Unfinished or Inactive

Pandemic Mitigation Optimization

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Contents

1 Model **2**

1.1 SIRD Model with Variable-Cost Interventions . . . 2

1.2 SIRD Model with Non-Variable-Cost Interventions . . . 4

1.3 Parameters in Trials . . . 4

2 Heuristics **4**

2.1 Lagrangian Heuristic and Lower Bound . . . 4

2.2 Index Policy . . . 7

2.3 w-Period *Time-Greedy* Heuristic . . . 9

2.4 w-Period *Time-Greedy* Heuristic with T^+-period Lookahead . . . 9

2.5 B-Policy *Policy-Greedy* Solution (implemented) . . . 10

2.6 Index and Assortment Index Policy . . . 11

2.7 Local Search . . . 11

2.8 F-Factor Early Stopping Using BARON/Gurobi/DICOPT/BONMIN . . . 11

2.9 Quadratic Policy Cost Approximation . . . 12

3 Results **16**

3.1 Comparison of Heuristics . . . 16

3.2 Lagrangian Heuristic Lower Bound Improvement . . . 22

3.2.1 Summaries of Results . . . 27

3.3 Lagrangian Subproblem Quasiconvexity . . . 27

4 Conclusion **29**

1 Model

1.1 SIRD Model with Variable-Cost Interventions

Parameters

- Suppose there are N total individuals, of whom I_0 are initially infected. There are m interventions to consider, each of which has (up to) n levels of intensity.
- Let A_{ijt} denote the fixed cost of implementing policy i at level j at time t.
- Let B_{ijt} denote the *switching* cost of implementing policy i at level j at time t (only incurred if policy not implemented in previous period).
- Let C_{ijt} denote the per-susceptible-individual cost of implementing poilcy i at level j at time t.
- Let $C_{infection}$ and C_{death} denote the costs associated with a single individual being infected in a given period, and a single individual losing their life due to disease, respectively.
- Let K_I correspond to the infection rate such that the number of new infections is proportional to K_I multiplied by the number of interactions between susceptible and infected individuals, modeled as the product of the sizes of those populations.
- Let K_R and K_D denote the proportion of infected individuals in each period who recover and die, respectively.
- Let P_{ijt} denote the factor by which new infections are decreased in period t as a result of implementing policy i at level j. In this model, these factors are independent of one another should multiple policies be implemented simultaneously.

Decision Variables

- Let
$$y_{ijt} = \begin{cases} 1 & : \text{policy } i \text{ is implemented at level } j \text{ in time period } t \\ 0 & : \text{otherwise} \end{cases} \tag{8a}$$
- Let
$$z_{ijt} = \begin{cases} 1 & : \begin{array}{l} \text{policy } i \text{ is implemented at level } j \text{ in time period } t, \\ \text{but not } t-1 \end{array} \\ 0 & : \text{otherwise} \end{cases} \tag{9a}$$

State Variables

- Let S_t, I_t, R_t, d_t, and D_t denote the population of individuals at time t who are Susceptible, Infected, Recovered, dying (in the current period), and Dead (cumulatively), respectively. These values depend on the interventions applied.

- Let P_t denote the cumulative factor by which new infections are decreased between periods $t-1$ and t. That is,

$$P_t = \prod_{\substack{i,j \text{ s.t.} \\ \text{policy } i \text{ used} \\ \text{at level } j \\ \text{in period } t}} P_{ijt} \tag{6a}$$

Model Formulation: Disease Mitigation Optimization (DMO)

$$\underset{y,P,S,I,R,D,d}{\text{Minimize}} \quad \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{t=1}^{T} A_{ijt}y_{ijt} + B_{ijt}z_{ijt} + C_{ijt}S_t y_{ijt} + \sum_{t=1}^{T} C_{infection}I_t + C_{death}d_t \tag{0}$$

$$\begin{aligned}
\text{s.t.} \quad & S_t = S_{t-1} - K_I \cdot P_t \cdot S_{t-1} \cdot I_{t-1} && \forall\, t \in \{2,\dots,T\} && (1)\\
& I_t = I_{t-1} + K_I \cdot P_t \cdot S_{t-1} \cdot I_{t-1} - K_R \cdot I_{t-1} - K_D \cdot I_{t-1} && \forall\, t \in \{2,\dots,T\} && (2)\\
& R_t = R_{t-1} + K_R \cdot I_{t-1} && \forall\, t \in \{2,\dots,T\} && (3)\\
& d_t = K_D \cdot I_{t-1} && \forall\, t \in \{2,\dots,T\} && (4)\\
& D_t = D_{t-1} + d_t && \forall\, t \in \{2,\dots,T\} && (5)\\
& P_t = \prod_{i=1}^{m}\prod_{j=1}^{n} (1 - y_{ijt} + P_{ijt} \cdot y_{ijt}) && \forall\, t \in \{1,\dots,T\} && (6)\\
& \sum_{j=1}^{n} y_{ijt} \leq 1 && \forall\, i,t && (7)\\
& y_{ijt} \in \{0,1\} && \forall\, i,j,t && (8)\\
& z_{ijt} \geq y_{ij(t)} - y_{ij(t-1)} \quad (\text{let } y_{ij0} = 0 \ \forall\, i,j) && \forall\, i,j,t && (9)\\
& 0 \leq z_{ijt} \leq 1 && \forall\, i,j,\ \forall\, t \geq 1 && (10)\\
& I_1 = I_0 \\
& S_1 = N - I_0 \\
& D_1 = 0 \\
& R_1 = 0 \\
& d_1 = 0
\end{aligned}$$

The objective function (0) is the sum of the costs of implementing the policy interventions in all periods and the costs associated with the resulting disease and death in all periods (due to lost productivity and resources).

The constraints (1),(2),(3),(4), and (5) model the SIRD compartment subpopulations as the disease progresses alongside the infection-reduction factors P_t at each period $t = 1, \dots, T$. The constraint (6) models the multiplicative effect of multiple interventions being applied in the same period, as described by (6a). Equation (7) enforces the logical constraint that at most one level from each policy be used in each period, and (8) ensures that the y_{ijt} variables correspond to the binary definition in (8a).

1.2 SIRD Model with Non-Variable-Cost Interventions

Replace the objective (0) in the mathematical program formulation of the **(DMO)** model with

$$\underset{y,P,S,I,R,D,d}{\text{Minimize}} \quad \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{t=1}^{T} A_{ijt}y_{ijt} + B_{ijt}z_{ijt} + C_{ijt}\cdot S_t \cdot y_{ijt} + \sum_{t=1}^{T} C_{infection}\cdot I_t + C_{death}\cdot d_t \quad (0)$$

$$\Downarrow$$

$$\underset{y,P,S,I,R,D,d}{\text{Minimize}} \quad \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{t=1}^{T} A_{ijt}y_{ijt} + B_{ijt}z_{ijt} + C_{ijt}\cdot N \cdot y_{ijt} + \sum_{t=1}^{T} C_{infection}\cdot I_t + C_{death}\cdot d_t \quad (11)$$

1.3 Parameters in Trials

All the numerical experiments performed utilized the following parameters:

- $N = 300000$
- $I_0 = 1000$
- $C_{infection} = 10000$
- $C_{death} = 10000000$
- $K_I = 6e-07$
- $K_R = 0.03$
- $K_D = 0.015$
- Policies:

Name	# Levels	P	A	B	C
1. "Movement"	2	[.95, .925]	[5e5, 1e6]	[1e6, 2e6]	[1e6, 2e6]
2. "Education (University level)"	2	[.995, .95]	[0, 0]	[0, 0]	[0, 0]
3. "Social Gatherings (in a house)"	4	[.995, .99, .975, .925]	[0, 0, 0, 0]	[0, 0, 0, 0]	[0, 0, 0, 0]
4. "Non-Food Service (bank,retail, etc)"	3	[.995, .95, .925]	[2.5e5, 5e5, 1e6]	[5e5, 1e6, 2e6]	[5e5, 1e6, 2e6]
5. "Restaurants"	2	[.95, .925]	[5e5, 1e6]	[1e6, 2e6]	[1e6, 2e6]
6. "Masking"	3	[.995, .95, .925]	[0, 0, 0]	[0, 0, 0]	[0, 0, 0]
7. "Mega Events"	3	[.995, .95, .925]	[2.5e5, 5e5, 1e6]	[5e5, 1e6, 2e6]	[5e5, 1e6, 2e6]
8. "Border Control"	2	[.95, .925]	[5e5, 1e6]	[1e6, 2e6]	[1e6, 2e6]
9. "Physical Distancing"	1	[.925]	[0]	[0]	[0]

Figure 1: *Policy parameters P_{ijt}, A_{ijt}, B_{ijt}, and C_{ijt} are in general dynamic (time-varying) in the full* **(DMO)** *model. In our numerical experiments, we use non-dynamic policies, and in each row of this table corresponding to policy i, parameters are listed for each level j.*

2 Heuristics

2.1 Lagrangian Heuristic and Lower Bound

We can utilize a Lagrangian relaxation to the full problem **(DMO)** by relaxing constraints (6) and instead penalizing the objective function using dual multipliers. The relaxed problem decomposes into two computationally less expensive subproblems; by iteratively updating the multipliers using gradient ascent, the lower bound tightens. Furthermore, we can transform a solution to the relaxed problem into a feasible solution for the full problem, yielding a heuristic solution in its own right.

First, we focus on the variant of the model in which interventions do not have a variable cost depending on the size of the susceptible population (S_t), and instead have a "variable" cost proportional to the size of the entire population (N), as in (11) (i.e., the costs do not "vary" between time periods). This allows the Lagrangian minimization problem to be split into two subproblems.

Then, we transform constraint (6) using a logarithm:

$$\ln P_t = \sum_{i=1}^{m}\sum_{j=1}^{n} \ln(1 - y_{ijt} + P_{ijt}y_{ijt}),\ \forall\ t = 1,\ldots,T. \tag{6-log}$$

Next, we remove this constraint from **(DMO)** and augment the objective (11) via multipliers $\lambda_t, t = 1,\ldots,T$ to obtain a relaxed minimization problem:

$$\underset{y,P,S,I,R,D,d}{\text{Minimize}} \quad [\text{Objective (11)}] + \sum_{t=1}^{T} \lambda_t \left(\sum_{i=1}^{m}\sum_{j=1}^{n} \ln(1 - y_{ijt} + P_{ijt}y_{ijt}) - \ln P_t \right) \tag{12}$$

$$\text{s.t.} \quad 0 \le P_t \le 1 \tag{13}$$

Constraints from **(DMO)** except for (6).

An optimal value to the augmented problem (12) is a lower bound to the optimal value of the full problem **(DMO)** . We add an extra constraint (13) to enforce the logical constraints that the policy effectiveness factors are between 0 and 1; note that in a numerical implementation, it may actually be preferable to precompute a reasonable lower bound on $\underline{P}_t \in (0,1)$ and constrain $\underline{P}_t \le P_t \le 1$ because the logarithm in (12) is undefined for $P_t = 0$.

By iteratively solving the augmented problem (12) and then using subgradient ascent to update λ_t for all $t = 1,\ldots,T$, we obtain increasingly tighter lower bounds on the optimal value for the full problem **(DMO)** .

Note that the augmented problem (12) can be decomposed into two minimization problems with optimal values $L_1(\boldsymbol{\lambda})$ and $L_2(\boldsymbol{\lambda})$:

$L_1(\boldsymbol{\lambda})$ is the solution to

$$\underset{y}{\text{Minimize}} \quad \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{t=1}^{T}[A_{ijt}\cdot y_{ijt} + B_{ijt}z_{ijt} + C_{ijt}\cdot N\cdot y_{ijt} + \lambda_t \ln(1 - y_{ijt} + P_{ijt}y_{ijt})]$$

$$\sum_{j=1}^{n} y_{ijt} \leq 1 \qquad \forall\, i,t \tag{7}$$

$$y_{ijt} \in \{0,1\} \qquad \forall\, i,j,t \tag{8}$$

$$z_{ijt} \geq y_{ij(t)} - y_{ij(t-1)} \quad \forall\, i,j,t \tag{9}$$

$$0 \leq z_{ijt} \leq 1 \qquad \forall\, i,j,\ \forall\, t \geq 1 \tag{10}$$

This integer program can be solved by standard off-the-shelf software such as Gurobi. Note: Nonlinear integer programs have only been solvable by Gurobi since November 2019. I wonder whether there is a more complete explanation of why this subproblem is in fact "easier" than the full problem.

$L_2(\boldsymbol{\lambda})$ is the solution to

$$\underset{P,S,I,R,D,d}{\text{Minimize}} \quad \sum_{t=1}^{T}[C_{infection}\cdot I_t + C_{death}\cdot d_t - \lambda_t \ln P_t]$$

$$\text{s.t.} \quad S_t = S_{t-1} - K_I\cdot P_t\cdot S_{t-1}\cdot I_{t-1} \qquad \forall\, t\in\{2,\ldots,T\} \tag{1}$$

$$I_t = I_{t-1} + K_I\cdot P_t\cdot S_{t-1}\cdot I_{t-1} - K_R\cdot I_{t-1} - K_D\cdot I_{t-1} \quad \forall\, t\in\{2,\ldots,T\} \tag{2}$$

$$R_t = R_{t-1} + K_R\cdot I_{t-1} \qquad \forall\, t\in\{2,\ldots,T\} \tag{3}$$

$$d_t = K_D\cdot I_{t-1} \qquad \forall\, t\in\{2,\ldots,T\} \tag{4}$$

$$D_t = D_{t-1} + d_t \qquad \forall\, t\in\{2,\ldots,T\} \tag{5}$$

$$P_t \leq 1 \tag{13}$$

$$I_1 = I_0$$

$$S_1 = N - I_0$$

$$D_1 = 0$$

$$R_1 = 0$$

$$d_1 = 0$$

This problem has no integer constraints and can be solved by any nonlinear programming software.

To increase the tightness of the bound in the gradient-ascent step for the multipliers $\boldsymbol{\lambda}$, where $\boldsymbol{\lambda}^+$ represents the vector of multipliers at a subsequent iteration, we use the updating rule:

$$\boldsymbol{\lambda}^+ = \boldsymbol{\lambda} + \gamma\left(\nabla L_1(\boldsymbol{\lambda}) + \nabla L_2(\boldsymbol{\lambda})\right),$$

i.e.

$$\lambda_t^+ = \lambda_t + \gamma \cdot \left(\sum_{i=1}^{m} \sum_{j=1}^{n} \ln(1 - y_{ijt} + P_{ijt} y_{ijt}) - \ln P_t \right). \tag{14}$$

To obtain a feasible solution to the full problem **(DMO)** after any iteration of this procedure, one can fix the values of $y_{ijt}, i = 1, \ldots, m, j = 1, \ldots, n, t = 1, \ldots, T$ in **(DMO)** to those obtained in the decomposed minimizations, which immediately yields values of $P_t, t = 1, \ldots, T$, which in turn gives values of the compartment subpopulations (S, I, R, d, and D) following basic bookkeeping.

This procedure can be iteratively performed indefinitely, and will yield a sequence of nondecreasing lower bounds to the full problem **(DMO)** . As a stopping criterion, one can terminate when the relative improvement between two iterations is less than a threshold ("improvement"), or when the relative optimality gap between the incumbent feasible solution and the greatest lower bound is less than a threshold ("optimality gap"). More formally, the heuristic corresponding to this procedure can be described as follows:

Initialize $\lambda_t \leftarrow 0$ for all t.
Do
    Minimize to obtain $L_1(\boldsymbol{\lambda})$ and $L_2(\boldsymbol{\lambda})$
    Update $\boldsymbol{\lambda}$ via gradient ascent as in (14)
    The value $L_1(\boldsymbol{\lambda}) + L_2(\boldsymbol{\lambda})$ gives a lower bound.
    The optimal values of y_{ijt} from $L_1(\boldsymbol{\lambda})$ substituted in the original problem **(DMO)**
    yield a feasible solution and thus an upper bound for that problem's optimal solution.
Repeat while stopping condition (improvement or optimality gap) is not met.

2.2 Index Policy

Index policies are natural heuristics and are likely to resemble intuitive decisionmaking in the absence of optimization tools. For comparison, we outline an index solution method and compare solutions produced in this manner and others in Section 3.1.

Consider a "block size" b, and the corresponding partition of time periods $\{1, \ldots, T\}$ into blocks of size b. For simplicity, we assume b divides T evenly:

$$\begin{aligned} B_1 &= (1, \ldots, b) \\ B_2 &= (b + 1, \ldots, 2b) \\ &\vdots \\ B_{T/b} &= (T - b + 1, T - b + 2, \ldots, T). \end{aligned}$$

Let $T^{B_k} = \max\{t | t \in B_1 \cup B_2 \cup \ldots \cup B_k\}$, i.e. the latest time period that appears in blocks $B_1, \ldots, B_k$, and let $t_0^{B_k} = \min\{t | t \in B_k\}$. For every block except possibly the last block, $T^{B_k} = k \cdot b$; for every block, $t_0^{B_k} = (k - 1)b + 1$.

A basic index policy is applied to all the periods in each block, one at a time, in order, using the index defined below in equation (15) for policy i at level j:

$$\text{index}(i, j, B) = P_{ijt} \times C_{ijt} \text{ (for some } t \text{ in block } B\text{.)} \tag{15}$$

[1]

For the following algorithm, we consider the $T_{horizon}$-period objective to be the objective obtained after $T_{horizon}$ periods, rather than the full T. We replace the objective of **(DMO)** (0) with the following:

$$\underset{y,P,S,I,R,D,d}{\text{Minimize}} \quad \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{t=1}^{T} A_{ijt}y_{ijt} + B_{ijt}z_{ijt} + C_{ijt}\cdot S_t \cdot y_{ijt} + \sum_{t=1}^{T} C_{infection}\cdot I_t \qquad + C_{death}\cdot d_t \tag{0}$$

$$\Downarrow$$

$$\underset{y,P,S,I,R,D,d}{\text{Minimize}} \quad \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{t=1}^{T_{horizon}} A_{ijt}y_{ijt} + B_{ijt}z_{ijt} + C_{ijt}\cdot S_t \cdot y_{ijt} + \sum_{t=1}^{T_{horizon}} C_{infection}\cdot I_t + C_{death}\cdot d_t \tag{16}$$

We refer to the variant of **(DMO)** with only $T_{horizon}$ time periods as $(\textbf{DMO})_{T_{horizon}}$. This algorithm iteratively focuses on $(\textbf{DMO})_{T^B}$ for each block $B = B_1, B_2, \ldots$.

We use the term "fix" to mean that a variable's value is set, and the variable is no longer treated as a decision variable but a parameter of the problem.

The index policy can be described as follows:

```
For each block B in {B_1, ..., B_{T/b}} in order,
    Fix y_ijt = 0 for all t ∈ B; do not change values for previous blocks.
    Set chosenPolicies← {}
    Do...
        Set oldObjectiveValue←objective of (DMO)_{T^B}
        Calculate (i*, j*) = arg min_{i,j} {index(i, j, B) : ∄j' s.t.(i, j') ∈ chosenPolicies}
        For each policy-level pair (i, j)
            If (i, j) ∈ chosenPolicies∪{(i*, j*)}
                Fix y_ijt = 1 for all t ∈ B
            Else
                Fix y_ijt = 0 for all t ∈ B
        Set newObjectiveValue←objective of (DMO)_{T^B}
        If newObjectiveValue < oldObjectiveValue
            Set chosenPolicies←chosenPolicies∪{(i*, j*)}
    ...repeat while newObjectiveValue < oldObjectiveValue
```

Note that this policy is both *greedy* with respect to policy decisions in each period in the sense

[1] The policy described here assumes that the cost and effectiveneess parameters, C_{ijt} and P_{ijt}, do not vary with respect to period t within blocks (note that in the numerical experiments in this paper, these parameters are not time-varying at all).

that the best indexed policies are chosen first, and also *myopic* with respect to time in the sense that only a small subset of time periods are considered in each iteration.

2.3 w-Period ***Time-Greedy*** Heuristic

The computational resources required to solve **(DMO)** to optimality will likely exceed what is available to decisionmakers when the number of policy options and the number of time periods are large. However, even with a large number of policy options, a small number of time periods may make the decision space small enough to solve to optimality even with an unsophisticated exhaustive search. Decisions made when considering a small number of time periods may be reasonable to use over a longer time horizon.

The 1-period *time-greedy* algorithm is the greedy policy in which decisions are made only considering one period at a time. In the w-period greedy heuristic, decisions are made only considering w periods at a time. After decisions have been made optimally over the first w periods, the decisions for period 1 are fixed, and the problem is solved for periods $2, \ldots, w+1$; on the l'th iteration, the horizon of optimization is $l, \ldots, l+w-1$. This continues until the horizon is $T-(w-1), \ldots, T$, for a total of $T-(w-1)$ iterations.

We refer to the variant of **(DMO)** with only $T_{horizon}$ time periods as $(\textbf{DMO})_{T_{horizon}}$, as in (16) in Section 2.2, and use the term "fix" in the same way as in Section 2.2.

```
Initialize $y_{ijt} \leftarrow 0$ for all i,j,t.
For $T_0$ in $\{1, \ldots, T-w+1\}$
    For all t<$T_0$
        Fix $y_{ijt}$ to whatever value it currently holds for all i,j.
        Fix $P_t, S_t, I_t, R_t, D_t, d_t$ to whatever values they currently hold.
    For all t$\in \{T_0, \ldots, T_0+w-1\}$
        Unfix $y_{ijt}$ for all i,j. Unfix $P_t, S_t, I_t, R_t, D_t, d_t$.
    Solve $(\textbf{DMO})_{T-w+1}$ with the un-fixed variables.
```

2.4 w-Period ***Time-Greedy*** Heuristic with T^+-period Lookahead

In the execution of the w-period time-greedy solution, and indeed any variant of **(DMO)** , the primary difficulty is optimally finding values of integer-constrained variables. It is trivial, in fact, to consider the disease progression over time periods subsequent to the w periods during which optimal interventions are being considered in the context of the w-period time-greedy heuristic. This motivates a lookahead heuristic, in which the quality of a decision is assessed not just on the disease-related and policy-related costs within a w-period interval, but additionally on the disease-related costs during T^+ subsequent time-periods, during which the decision-maker does not make *any* policy decisions (and so no interventions are chosen).

This should yield more aggressive policy decisions than the w-period time-greedy heuristic, as the disease-related costs associated with any policy intervention menu are higher, and thus it will be desirable to further decrease infections during the w-period window.

The difference between these two heuristics can be summarized as follows:

- In the w-period time-greedy heuristic, only w periods are considered at a time in terms of decisionmaking and disease progression.
- In the T^+-period lookahead variant, the decisionmaker's "hands are tied" (they are forced to use no intervention) after the w periods of decisionmaking, but they calculate and make decisions based on the costs associated with disease progression during an additional T^+ time periods.

The algorithm is as follows:

```
Initialize $y_{ijt} \leftarrow 0$ for all i,j,t.
For $T_0$ in $\{1, \ldots, T-w+1\}$
    For all t<$T_0$
        Fix $y_{ijt}$ to whatever value it currently holds for all i,j.
        Fix $P_t, S_t, I_t, R_t, D_t, d_t$ to whatever values they currently hold.
    For all t∈ $\{T_0, \ldots, T_0+w-1\}$
        Unfix $y_{ijt}$ for all i,j. Unfix $P_t, S_t, I_t, R_t, D_t, d_t$.
    Solve $(\textbf{DMO})_{T-w+1+T^+}$ with the un-fixed variables.
```

2.5 B-Policy ***Policy-Greedy*** Solution (implemented)

Much of the difficulty of solving **(DMO)** largely stems from the highly nonlinear constraint (6), which involves the product of $m \times n$ integer-constrained variables. On the other hand, if only a single policy with a single level is considered, the problem can be solved to optimality over a long time horizon quickly, with modest computational resources.

The following *policy-greedy* algorithm leverages this fact to make optimal decisions for only one policy at a time, fixing the plan for that policy while considering adding another, until either B policies are chosen or there is no improvement from adding any additional policy (at any level).

To articulate this, we introduce parameters P_t^0 for $t = 1, \ldots, T$, and modify constraint (6) to

$$P_t = \prod_{i=1}^{m} \prod_{j=1}^{n} (1 - y_{ijt} + P_{ijt} \cdot y_{ijt}) \ \ \forall\, t \in \{1, \ldots, T\} \tag{6}$$

$$\Downarrow$$

$$P_t = P_t^0 \prod_{i=1}^{m} \prod_{j=1}^{n} (1 - y_{ijt} + P_{ijt} \cdot y_{ijt}) \ \ \forall\, t \in \{1, \ldots, T\} \tag{17}$$

where P_t^0 (instead of simply "1") represents the factor by which the infection rate is decreased by decisionmaking *if no policies are implemented.*

```
Set USED← {}, OBJECTIVE← ∞
For b in {1,...,B}
    Set ITERATION_OBJECTIVE← ∞
    For each policy i
        If (i,j) ∉USED for any of j∈ {1,...,n}  # Policy i has not been used at any level
            For each level j
                Set m←1, n←1, and solve (DMO) with only policy i at level j.
                Solve this problem
                Set SOLUTION_OBJECTIVE←objective value of this problem
                If SOLUTION_OBJECTIVE<ITERATION_OBJECTIVE
                    Set ITERATION_OBJECTIVE←SOLUTION_OBJECTIVE
    If ITERATION_OBJECTIVE<OBJECTIVE
        Set OBJECTIVE←ITERATION_OBJECTIVE
        Set USED←USED∪{(i,j)}
        Set P_t^0 ← P_t^0 × P_ijt for all t where policy i at level j is used in the solution
        that produced ITERATION_OBJECTIVE
    Else
        Terminate without adding any new policy
```

2.6 Index and Assortment Index Policy

2.7 Local Search

2.8 F-Factor Early Stopping Using BARON/Gurobi/DICOPT/BONMIN

The following heuristic requires a solution strategy (a “solver”) for the **(DMO)** model formulated in Section 1.1 that can iteratively generate the following two quantities:

1. A sequence of feasible solutions with improving objective function values, referred to as “incumbent solutions” whose objective values serve as upper bounds for the problem, and

2. a sequence of increasing lower bounds for the problem, generated from any of the following:

    - continuous relaxation,
    - Lagrangian relaxation,
    - any other dualization or constraint relaxation.

This is, in fact, what most mathematical programming solvers aim to iteratively produce while solving a problem. We refer to a “solver” as a tool that achieves the two goals above. As the solvers compute, the percent difference between the lower and upper bounds - the “relative optimality gap” - shrinks. Mixed-integer programming tools typically do not prove optimality, but stop when this relative optimality gap falls below an acceptable threshold.

With an upper bound u and a lower bound l to the objective function, solving the problem to desired optimality factor F requires that

$$\frac{u-l}{u} < F.$$

Selecting a large value of F would amount to an "early-stopping" heuristic, and the solution may still be useful even though there is no reason to suspect that the generated solution is globally optimal.

```
Begin solving the (DMO) problem using a solver. For each iteration, do
    If (u-l)/u < F
        Stop
    Else
        Continue
```

2.9 Quadratic Policy Cost Approximation

The full **(DMO)** model considers atomic policy decisions and may be of great utility when there are too many possible policy decisions in each period for a policy-maker to evaluate them all via analysis or simulation. Unfortunately, in this context of a large number of policies (m) and levels (n), it becomes difficult to extract high-level insights from optimal decisions produced by such a model.

Furthermore, since we demonstrate that realistically large instances of the **(DMO)** model become intractible, effective high-level patterns in approximate solutions may be obscured by the "noise" of sub-optimal decisions.

An alternative, simplified model may be useful for analysis of the problem. Such a model, when fitted to real problem parameters, may also be useful to policy-makers for setting cost targets. One such simplification is to approximate all the costs associated with different policy assortments as smooth functions of an "effort level" that exactly corresponds to the policy "effectiveness" P_{ijt}.

Consider a policy assortment ω as a "policy vector"

$$\omega = \begin{bmatrix} j_{\omega 1} & \dots & j_{\omega n} \end{bmatrix},$$

where $j_{\omega i}$ is the level of policy i utilized in assortment ω. Denote the set of all possible policy assortments Ω. Note that in each period of the model **(DMO)** , exactly one policy assortment is utilized, and each policy assortment has associated "assortment parameters". That is, if assortment ω is used in period t, then the associated probability of becoming infected P_t can be called P_ω, and the associated setup and per-susceptible-individual costs A_ω and C_ω can be written as the sum of the costs of each atomic policy included in ω. The assotment $\mathbf{0}$ includes no policies, and has $P_{\mathbf{0}} = 1, A_{\mathbf{0}} = 0, C_{\mathbf{0}} = 0$.

Note that the switching costs of an assortment is not well-defined. The cost to switch from one assortment ω_1 to another ω_2 in the **(DMO)** model depends on which atomic policies are included in

ω_1 and ω_2. If they largely overlap, the switching cost will be small. Thus, an "assortment switching cost" B_ω is inappropriate conceptually unless it is ascribed a different meaning than in the **(DMO)** model[2].

Setting aside for the moment switching costs, consider that some policy assortments may be *dominated* by others. That is, for two assortments ω_1 and ω_2, it is possible that the infection probability factor $P_{\omega_1} < P_{\omega_2}$ and also the costs $A_{\omega_1} < A_{\omega_2}$ and $C_{\omega_1} < C_{\omega_2}$. If policy assortments are the unit of analysis, it may be desirable to consider only efficient assortments.

Given the policies described in Section 1.3, we demonstrate efficient policy assortments in Figure 2, along with a quadratic approximation of the costs associated with the efficient policies.

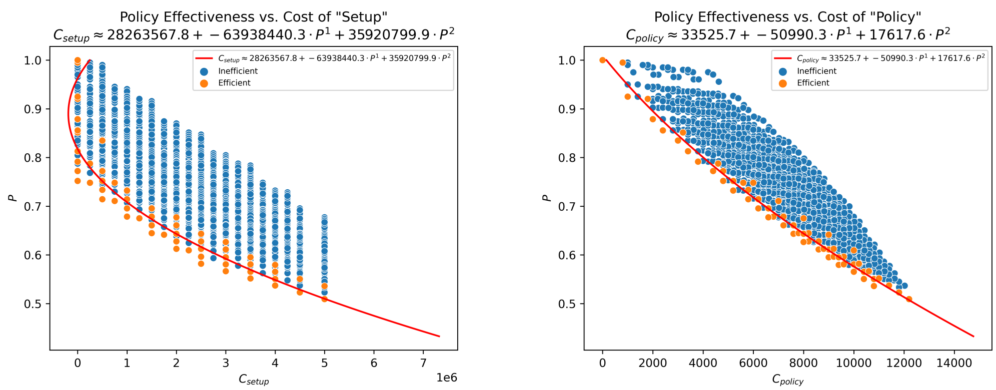


Figure 2: *From all possible assortments of the policies listed in Section 1.3, "efficient" assortments are approximated by a quadratic function of "effectiveness"* P_t.

Given the 9 atomic policies described in Section 1.3, each with between 1 and 4 levels, there are a total of $51,840$ possible policy assortments. Remarkably, only 61 of these assortments are non-dominated. For example, one of the "efficient" assortments in Figure 2 is "Movement (level 1/2) & Masking (level 2/3) & Physical Distancing (level 1/1)" parameters.

The **(DMO)** problem can in fact be solved as a bilinear program (rather than a nonlinear program including higher-order terms) by considering policy assortments as the unit of analysis, rather than atomic policies. Unfortunately, even when restricting to "efficient" assortments, this does not yield a significantly more tractable problem, as there are still likely to be a relatively high number of possible assortments and a long time horizon. However, a quadratic approximation of policy assortments may be tractable and may yield interpretable high-level output.

Suppose, as in Figure 2, the policy effectiveness of each assortment is calculated, and the setup and per-susceptible-individual costs of utilizing each assortment ω in period t are approximated as follows:

[2] It may be possible to include a policy regulation term that penalizes large period-to-period changes in policy, such as $\sum_{t=2}^{T}(P_t - P_{t-1})^2$.

- If assortment ω is used in period t, then the policy effectiveness in the period is

$$P_t = P^\omega = \prod_{\substack{\{(i,j):\text{policy } i,\\ \text{level } j \text{ in } \omega\}}} P_{ijt}. \tag{18}$$

  There is a "maximally effective" policy assortment consisting of every atomic policy at the highest possible level. This assortment is always non-dominated, and its effectiveness p_{min} serves as a lower bound for policy effectiveness: $p_{min} \leq P_t \leq 1$

- If assortment ω is used in period t, then the setup cost in the period is

$$A_t = A^\omega = \sum_{\substack{\{(i,j):\text{policy } i,\\ \text{level } j \text{ in } \omega\}}} A_{ijt} \tag{19}$$

  which we approximate as as "$A(P_t)$":

$$A_t \approx a_2 P_t^2 + a_1 P_t + a_0$$

- If assortment ω is used in period t, then the per-susceptible-individual cost in the period is

$$C_t = C^\omega = \sum_{\substack{\{(i,j):\text{policy } i,\\ \text{level } j \text{ in } \omega\}}} C_{ijt} \tag{20}$$

  which we approximate as "$C(P_t)$":

$$C_t \approx c_2 P_t^2 + c_1 P_t + c_0$$

The following mathematical program describes this approximation (note the absence of switching costs):

$$
\begin{aligned}
\underset{P,A,C,S,I,R,D,d}{\text{Minimize}} \quad & \sum_{t=1}^{T} A_t + C_t S_t + \sum_{t=1}^{T} C_{infection} I_t \quad + C_{death} d_t && (21)\\
\text{s.t.} \quad & S_t = S_{t-1} - K_I \cdot P_t \cdot S_{t-1} \cdot I_{t-1} && \forall\, t \in \{2,\dots,T\}\\
& I_t = I_{t-1} + K_I \cdot P_t \cdot S_{t-1} \cdot I_{t-1} - K_R \cdot I_{t-1} - K_D \cdot I_{t-1} && \forall\, t \in \{2,\dots,T\}\\
& R_t = R_{t-1} + K_R \cdot I_{t-1} && \forall\, t \in \{2,\dots,T\}\\
& d_t = K_D \cdot I_{t-1} && \forall\, t \in \{2,\dots,T\}\\
& D_t = D_{t-1} + d_t && \forall\, t \in \{2,\dots,T\}\\
& A_t = a_2 P_t^2 + a_1 P_t + a_0 && \forall\, t \in \{1,\dots,T\}\\
& C_t = c_2 P_t^2 + c_1 P_t + c_0 && \forall\, t \in \{1,\dots,T\}\\
& p_{min} \leq P_t \leq 1 && \forall\, t \in \{1,\dots,T\}\\
& I_1 = I_0\\
& S_1 = N - I_0\\
& D_1 = 0\\
& R_1 = 0\\
& d_1 = 0
\end{aligned}
$$

A solution to problem (21) yields a natural heuristic solution method for the **(DMO)** mode (0) by finding policy assortments that yield similar cost and probability decisions. Specifically, for any value of (A_t, C_t, P_t), let $\omega(A_t, C_t, P_t)$ be the policy assortment that yields the "closest" values $(A^\omega, C^\omega, P^\omega)$ as defined by equations (18), (19), and (20) and some loss function L. Consider the loss function

$$L((A,C,P),\omega) = \frac{A^\omega - A}{A+\epsilon} + \frac{C^\omega - C}{C+\epsilon} + \frac{P^\omega - P}{P+\epsilon} \tag{22}$$

and then the "closest" policy is defined as

$$\omega(A,C,P) = \underset{\omega\in\Omega}{\arg\min}\, L((A,C,P),\omega). \tag{23}$$

Finally, recalling that $j_{\omega i}$ is the level of policy i implied by assortment ω, define the mapping of a policy assortment ω to a binary decision of whether intervention i is used at level j as:

$$y_{ij}^{\omega} = \begin{cases} 1 & : j = j_{\omega i} \\ 0 & : \text{otherwise} \end{cases}. \tag{24}$$

The heuristic solution method utilizing solutions of the quadratically approximated problem (21) can then be written as follows:

Solve approximated problem (21) to obtain A_t, C_t, P_t for all t.
Calculate the ``closest'' assortment for each period $\omega_t = \omega(A_t, C_t, P_t)$
Use the policy decisions in the full **(DMO)** problem by substituting $y_{ijt} = y_{ij}^{\omega_t}$

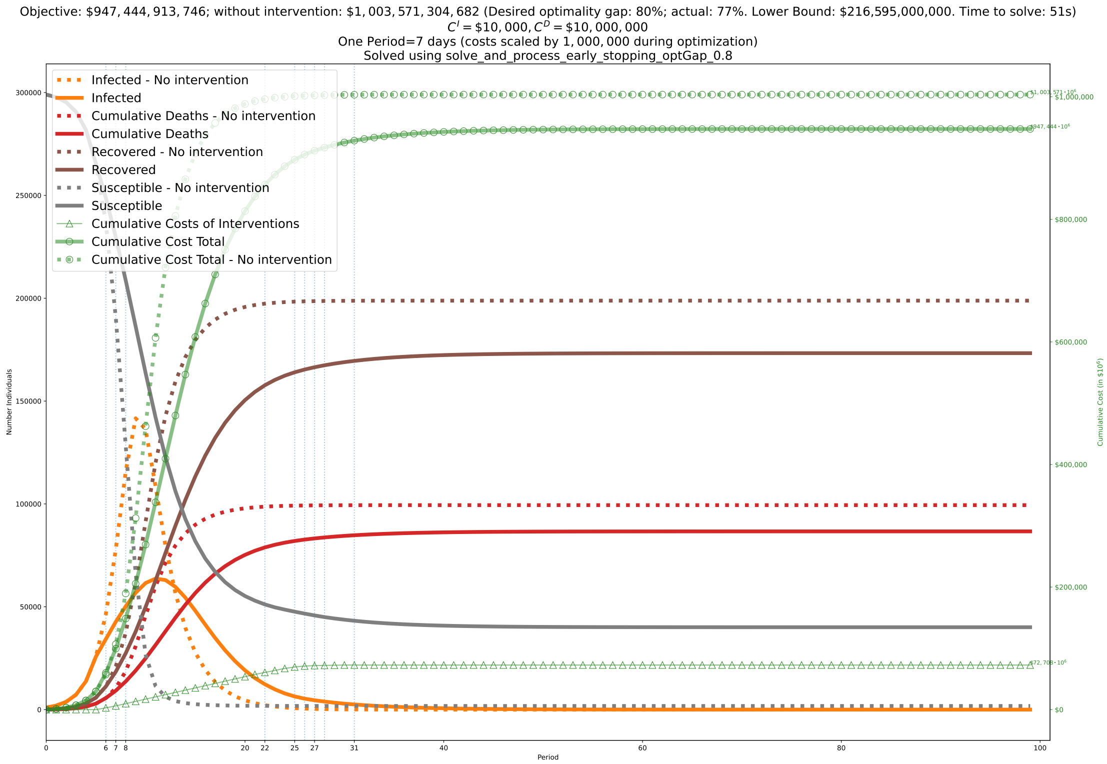
3 Results

3.1 Comparison of Heuristics

Here we present three solutions for the same **(DMO)** instance with $T = 100$ periods, calculated using different heuristics. The first is the “F-Factor Early Stopping” heuristic defined in Section 2.8 with $F = 0.8$; the second is the Lagrangian heuristic defined in Section 2.1; the third is the quadratic heuristic defined in Section 2.9.

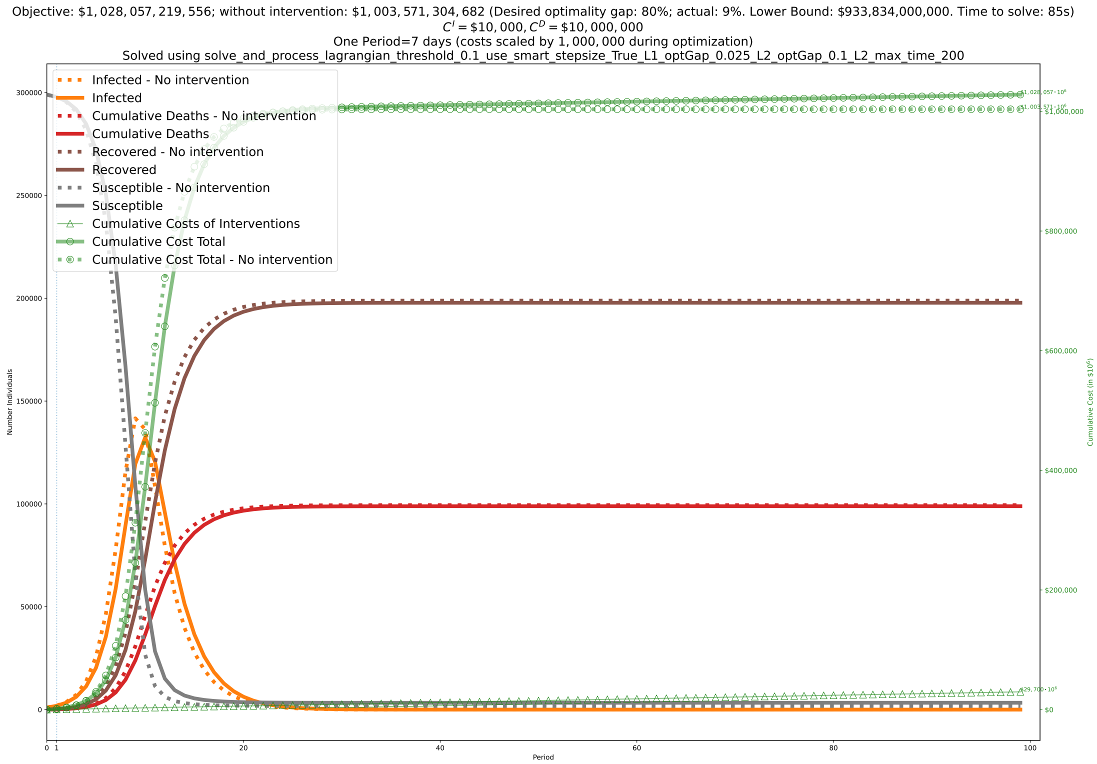
Note that the early-stopping heuristic in Figure 3a generates the best solution overall, the Lagrangian solution in Figure 3b, utilizing the Lagrangian lower-bound, has the tightest lower-bound[3], and the quadratic approximation in Figure 3c, which is solved by ignoring switching costs, has the most policy changes over the horizon.

[3] the listed lower bound of the quadratic approximation is not a true lower bound, but a lower bound of the quadratic approximation (21)



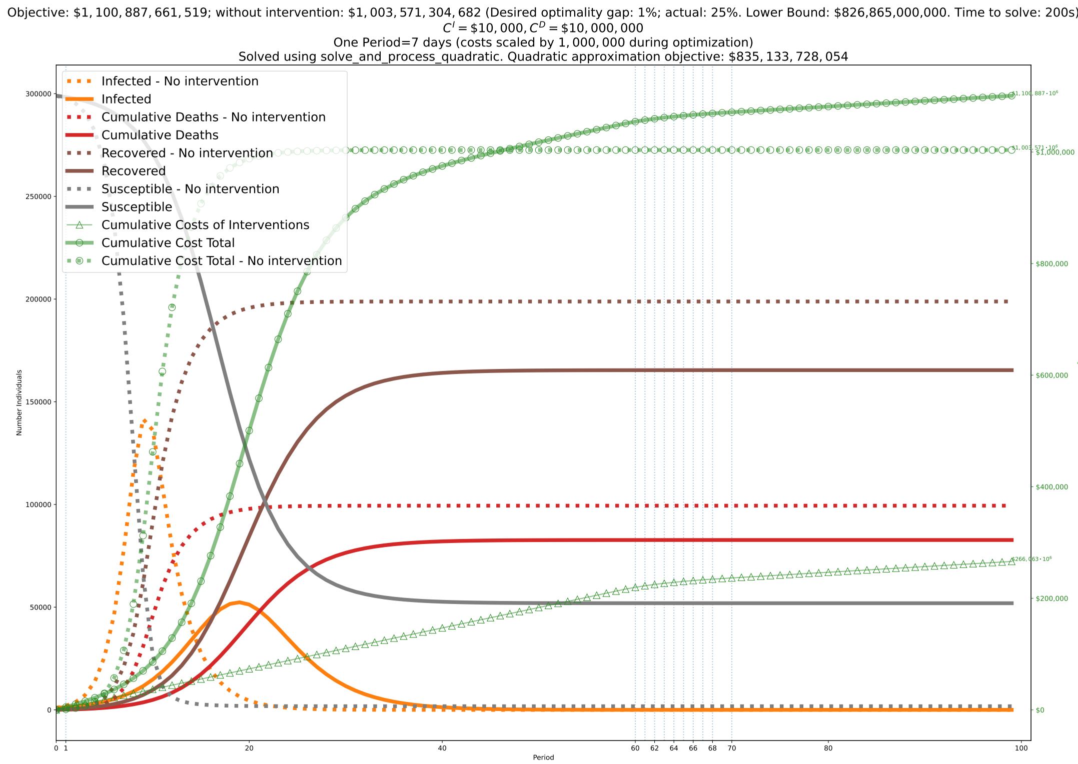
	0 -5	6 -6	7 -7	8 -21	22 -24	25 -25	26 -26	27 -27	28 -30	31 -99
0. Movement A: $[5000 ,10000]·10² B: $[10000,20000]·10² C: $[10 ,14]·10² P: [.95 ,.93]		2	2	2	2					
1. Education (University level) A: $[0 ,0]·10² B: $[0 ,0]·10² C: $[10 ,14]·10² P: [.99 ,.95]				2						
2. Social Gatherings (in a house) A: $[0 ,0 ,0 ,0]·10² B: $[0 ,0 ,0 ,0]·10² C: $[8 ,10 ,12 ,14]·10² P: [.99 ,.99 ,.97 ,.93]		4	4	4	4	4	4	4		
3. Non-Food Service (bank,retail, etc) A: $[2500 ,5000 ,10000]·10² B: $[5000 ,10000,20000]·10² C: $[8 ,10 ,14]·10² P: [.99 ,.95 ,.93]		3	3	3	3	3				
4. Restaurants A: $[5000 ,10000]·10² B: $[10000,20000]·10² C: $[10 ,14]·10² P: [.95 ,.93]			2	2	2	2	2			
5. Masking A: $[0 ,0 ,0]·10² B: $[0 ,0 ,0]·10² C: $[8 ,10 ,14]·10² P: [.99 ,.95 ,.93]		3	3	3	3	3	3			
6. Mega Events A: $[2500 ,5000 ,10000]·10² B: $[5000 ,10000,20000]·10² C: $[8 ,10 ,14]·10² P: [.99 ,.95 ,.93]		3	3	3	3	3				
7. Border Control A: $[5000 ,10000]·10² B: $[10000,20000]·10² C: $[10 ,14]·10² P: [.95 ,.93]		2	2	2	2					
8. Physical Distancing A: $[0]·10² B: $[0]·10² C: $[10]·10² P: [.93]		1	1	1	1	1	1	1	1	
Cost Per Period: TOTAL Cost Per Period: POLICY Cost Per Period: DISEASE Probability Factor	$4.9e+09 $0.0 $4.9e+09 1.000	$3e+10 $2.8e+ $2.7e+ 0.579	$4e+10 $3.2e+ $3.6e+ 0.536	$5.3e+10 $3.7e+09 $4.9e+10 0.509	$1.6e+10 $3.2e+09 $1.3e+10 0.536	$1.1e+ $2.4e+ $8.3e+ 0.626	$8.3e+ $1.6e+ $6.8e+ 0.732	$6.3e+ $7.2e+ $5.6e+ 0.856	$4.5e+09 $3e+08 $4.2e+09 0.925	$3.2e+08 $0.0 $3.2e+08 1.000

(a) *Solution using* 0.8*-Factor Early Stopping heuristic with the BARON solver.*



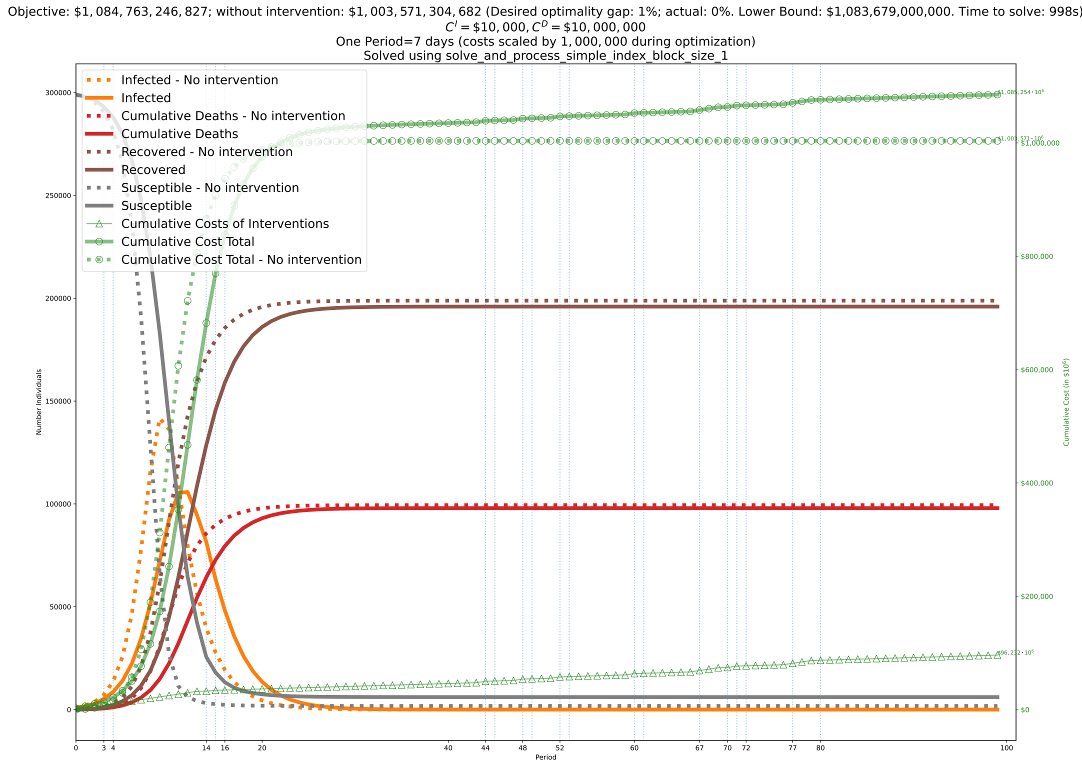
	1 -99
0. Movement A: $[5000 ,10000]·$10^2$ B: $[10000,20000]·$10^2$ C: $[10 ,14]·$10^2$ P: [.95 ,.93]	
1. Education (University level) A: $[0 ,0]·$10^2$ B: $[0 ,0]·$10^2$ C: $[10 ,14]·$10^2$ P: [.99 ,.95]	
2. Social Gatherings (in a house) A: $[0 ,0 ,0 ,0]·$10^2$ B: $[0 ,0 ,0 ,0]·$10^2$ C: $[8 ,10 ,12 ,14]·$10^2$ P: [.99 ,.99 ,.97 ,.93]	
3. Non-Food Service (bank,retail, etc) A: $[2500 ,5000 ,10000]·$10^2$ B: $[5000 ,10000,20000]·$10^2$ C: $[8 ,10 ,14]·$10^2$ P: [.99 ,.95 ,.93]	
4. Restaurants A: $[5000 ,10000]·$10^2$ B: $[10000,20000]·$10^2$ C: $[10 ,14]·$10^2$ P: [.95 ,.93]	
5. Masking A: $[0 ,0 ,0]·$10^2$ B: $[0 ,0 ,0]·$10^2$ C: $[8 ,10 ,14]·$10^2$ P: [.99 ,.95 ,.93]	
6. Mega Events A: $[2500 ,5000 ,10000]·$10^2$ B: $[5000 ,10000,20000]·$10^2$ C: $[8 ,10 ,14]·$10^2$ P: [.99 ,.95 ,.93]	
7. Border Control A: $[5000 ,10000]·$10^2$ B: $[10000,20000]·$10^2$ C: $[10 ,14]·$10^2$ P: [.95 ,.93]	
8. Physical Distancing A: $[0]·$10^2$ B: $[0]·$10^2$ C: $[10]·$10^2$ P: [.93]	1
Cost Per Period: TOTAL Cost Per Period: POLICY Cost Per Period: DISEASE Probability Factor	$1e+10 $3e+08 $1e+10 0.925

(b) *Solution using the Lagrangian heuristic.*



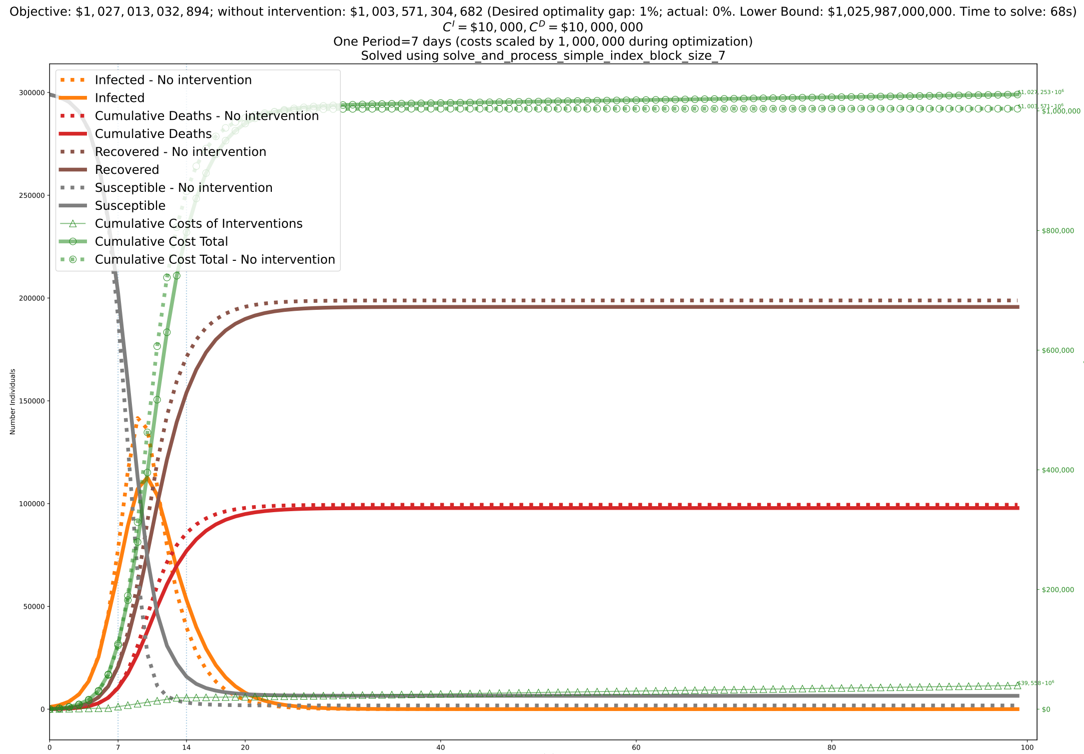
	1 -59	60 -60	61 -61	62 -62	63 -63	64 -64	65 -65	66 -66	67 -67	68 -69	70 -99
0. Movement A: $[5000 ,10000]$\cdot 10^2$ B: $[10000,20000]$\cdot 10^2$ C: $[10 ,14]$\cdot 10^2$ P: [.95 ,.93]	2	2	1	1	1	1	1	1			
1. Education (University level) A: $[0 ,0]$\cdot 10^2$ B: $[0 ,0]$\cdot 10^2$ C: $[10 ,14]$\cdot 10^2$ P: [.99 ,.95]	2	2					2				
2. Social Gatherings (in a house) A: $[0 ,0 ,0 ,0]$\cdot 10^2$ B: $[0 ,0 ,0 ,0]$\cdot 10^2$ C: $[8 ,10 ,12 ,14]$\cdot 10^2$ P: [.99 ,.99 ,.97 ,.93]	4	4	4	4	4	4	4	4	4	4	4
3. Non-Food Service (bank,retail, etc) A: $[2500 ,5000 ,10000]$\cdot 10^2$ B: $[5000 ,10000,20000]$\cdot 10^2$ C: $[8 ,10 ,14]$\cdot 10^2$ P: [.99 ,.95 ,.93]	3	3	2	2	2	2			1		
4. Restaurants A: $[5000 ,10000]$\cdot 10^2$ B: $[10000,20000]$\cdot 10^2$ C: $[10 ,14]$\cdot 10^2$ P: [.95 ,.93]	2	2	1	1	1						
5. Masking A: $[0 ,0 ,0]$\cdot 10^2$ B: $[0 ,0 ,0]$\cdot 10^2$ C: $[8 ,10 ,14]$\cdot 10^2$ P: [.99 ,.95 ,.93]	3	3	3	3	3	3	2	3	3	3	2
6. Mega Events A: $[2500 ,5000 ,10000]$\cdot 10^2$ B: $[5000 ,10000,20000]$\cdot 10^2$ C: $[8 ,10 ,14]$\cdot 10^2$ P: [.99 ,.95 ,.93]	3	3	3	2							
7. Border Control A: $[5000 ,10000]$\cdot 10^2$ B: $[10000,20000]$\cdot 10^2$ C: $[10 ,14]$\cdot 10^2$ P: [.95 ,.93]	2		1								
8. Physical Distancing A: $[0]$\cdot 10^2$ B: $[0]$\cdot 10^2$ C: $[10]$\cdot 10^2$ P: [.93]	1	1	1	1	1	1	1	1	1	1	1
Cost Per Period: TOTAL **Cost Per Period: POLICY** **Cost Per Period: DISEASE** **Probability Factor**	$1.8e+10 $3.7e+09 $1.4e+10 0.509	$3.3e+ $3.2e+ $1.8e+ 0.550	$2.8e+ $2.8e+ $1.4e+ 0.596	$2.4e+ $2.3e+ $1.2e+ 0.645	$2.1e+ $2e+09 $9.7e+ 0.679	$1.7e+ $1.7e+ $8.1e+ 0.714	$1.7e+ $1.7e+ $6.8e+ 0.734	$1.4e+ $1.4e+ $5.8e+ 0.752	$1.4e+ $1.4e+ $4.9e+ 0.787	$1.1e+ $1.1e+ $3.9e+ 0.791	$1e+09 $1e+09 $7.4e+05 0.813

(c) *Solution using the quadratic approximation. Note that the lower bound listed is a lower bound on the quadratic approximation* (21)*, not the full* **(DMO)** *problem.*



	0 -2	3 -3	4 -13	14 -14	15 -15	16 -43	44 -44	45 -47	48 -48	49 -51	52 -52	53 -59	60 -60	61 -66	67 -69	70 -70	71 -71	72 -76	77 -79	80 -99
0. Movement A: $[5000 ,10000]·$10^2$ B: $[10000,20000]·$10^2$ C: $[10 ,14]·$10^2$ P: [.95 ,.93]	1		1		1		1		1		1		1		1		1		1	
1. Education (University level) A: $[0 ,0]·$10^2$ B: $[0 ,0]·$10^2$ C: $[10 ,14]·$10^2$ P: [.99 ,.95]	1		1				1		1		1		1		1		1		1	
2. Social Gatherings (in a house) A: $[0 ,0 ,0 ,0]·$10^2$ B: $[0 ,0 ,0 ,0]·$10^2$ C: $[8 ,10 ,12 ,14]·$10^2$ P: [.99 ,.99 ,.97 ,.93]	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3. Non-Food Service (bank,retail, etc) A: $[2500 ,5000 ,10000]·$10^2$ B: $[5000 ,10000,20000]·$10^2$ C: $[8 ,10 ,14]·$10^2$ P: [.99 ,.95 ,.93]	1		1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4. Restaurants A: $[5000 ,10000]·$10^2$ B: $[10000,20000]·$10^2$ C: $[10 ,14]·$10^2$ P: [.95 ,.93]	1		1				1		1		1		1		1		1		1	
5. Masking A: $[0 ,0 ,0]·$10^2$ B: $[0 ,0 ,0]·$10^2$ C: $[8 ,10 ,14]·$10^2$ P: [.99 ,.95 ,.93]	1		1		1		1		1		1		1		1		1		1	
6. Mega Events A: $[2500 ,5000 ,10000]·$10^2$ B: $[5000 ,10000,20000]·$10^2$ C: $[8 ,10 ,14]·$10^2$ P: [.99 ,.95 ,.93]	1		1		1		1		1		1		1		1		1		1	
7. Border Control A: $[5000 ,10000]·$10^2$ B: $[10000,20000]·$10^2$ C: $[10 ,14]·$10^2$ P: [.95 ,.93]	1		1				1		1		1		1		1		1		1	
8. Physical Distancing A: $[0]·$10^2$ B: $[0]·$10^2$ C: $[10]·$10^2$ P: [.93]	1		1		1		1		1		1		1		1		1		1	
Cost Per Period: TOTAL Cost Per Period: POLICY Cost Per Period: DISEASE Probability Factor	$3.4e+09 $2.5e+09 $9.5e+08 0.773	$3.2e+ $2.4e+ $2.9e+ 0.995	$5.7e+10 $2.5e+09 $5.4e+10 0.773	$1e+11 $2.4e+ $1e+11 0.995	$8.8e+ $1.6e+ $8.6e+ 0.861	$9.5e+09 $4.8e+08 $9e+09 0.990	$2.5e+ $2.5e+ $5.6e+ 0.773	$4.8e+08 $4.8e+08 $2.9e+06 0.990	$2.5e+ $2.5e+ $1.4e+ 0.773	$4.8e+08 $4.8e+08 $7.4e+05 0.990	$2.5e+ $2.5e+ $3.6e+ 0.773	$4.8e+08 $4.8e+08 $1.1e+05 0.990	$2.5e+ $2.5e+ $2.3e+ 0.773	$4.8e+08 $4.8e+08 $6.9e+03 0.990	$2.5e+09 $2.5e+09 $4.7 0.773	$4.8e+ $4.8e+ $0.0 0.990	$2.5e+ $2.5e+ $0.0 0.773	$4.8e+08 $4.8e+08 $0.0 0.990	$2.5e+09 $2.5e+09 $0.0 0.773	$4.8e+08 $4.8e+08 $0.0 0.990

(d) *Solution using the index policy described in Section 2.2, using a block size of $b = 1$.*



	0 -6	7 -13	14 -99
0. Movement A: $[5000 ,10000]·10^2 B: $[10000,20000]·10^2 C: $[10 ,14]·10^2 P: [.95 ,.93]		1	
1. Education (University level) A: $[0 ,0]·10^2 B: $[0 ,0]·10^2 C: $[10 ,14]·10^2 P: [.99 ,.95]		1	
2. Social Gatherings (in a house) A: $[0 ,0 ,0 ,0]·10^2 B: $[0 ,0 ,0 ,0]·10^2 C: $[8 ,10 ,12 ,14]·10^2 P: [.99 ,.99 ,.97 ,.93]	1	1	1
3. Non-Food Service (bank,retail, etc) A: $[2500 ,5000 ,10000]·10^2 B: $[5000 ,10000,20000]·10^2 C: $[8 ,10 ,14]·10^2 P: [.99 ,.95 ,.93]		1	
4. Restaurants A: $[5000 ,10000]·10^2 B: $[10000,20000]·10^2 C: $[10 ,14]·10^2 P: [.95 ,.93]		1	
5. Masking A: $[0 ,0 ,0]·10^2 B: $[0 ,0 ,0]·10^2 C: $[8 ,10 ,14]·10^2 P: [.99 ,.95 ,.93]		1	
6. Mega Events A: $[2500 ,5000 ,10000]·10^2 B: $[5000 ,10000,20000]·10^2 C: $[8 ,10 ,14]·10^2 P: [.99 ,.95 ,.93]		1	
7. Border Control A: $[5000 ,10000]·10^2 B: $[10000,20000]·10^2 C: $[10 ,14]·10^2 P: [.95 ,.93]		1	
8. Physical Distancing A: $[0]·10^2 B: $[0]·10^2 C: $[10]·10^2 P: [.93]		1	
Cost Per Period: TOTAL Cost Per Period: POLICY Cost Per Period: DISEASE Probability Factor	$8.3e+09 $2.4e+08 $8.1e+09 0.995	$9.5e+10 $2.5e+09 $9.3e+10 0.773	$3.5e+09 $2.4e+08 $3.3e+09 0.995

(e) *Solution using the index policy described in Section 2.2, using a block size of $b = 7$.*

3.2 Lagrangian Heuristic Lower Bound Improvement

The lower bound on the objective value of **(DMO)** obtained by optimizing the (decomposed) Lagrangian relaxation described in Section 2.1 seems relatively tight on several problem instances. The corresponding heuristic also performs quite well. On the other hand, the BARON solver is able to generate solutions within a few minutes to the full **(DMO)** problem that are extremely high quality, but it does not guarantee a reasonable level of optimality even after running for hours. In particular, the BARON solver generates lower bounds as part of its numerical optimization procedure, but these lower bounds are nowhere near the values it obtains. The bounds generated by the Lagrangian procedure prove that these solutions are nearly optimal. This yields a stopping condition for the BARON solver that guarantees a desired level of optimality.

The following trials were considered to evaluate the performance of the Lagrangian method:

Trial	T	cost multiplier	effect multiplier	m	n	nConstraints	nPolicies	nVariables
0	20	1.0	0.5	9	4	1015	51840	1560
1	20	1.0	0.6	9	4	1015	51840	1560
2	20	1.0	0.7	9	4	1015	51840	1560
3	20	1.0	0.8	9	4	1015	51840	1560
4	20	1.0	0.9	9	4	1015	51840	1560
5	20	1.0	1.0	9	4	1015	51840	1560
6	20	1.0	1.1	9	4	1015	51840	1560
7	20	1.0	1.2	9	4	1015	51840	1560
8	20	1.0	1.3	9	4	1015	51840	1560
9	20	1.0	1.4	9	4	1015	51840	1560
10	20	1.0	1.5	9	4	1015	51840	1560
11	20	0.5	1.0	9	4	1015	51840	1560
12	20	0.6	1.0	9	4	1015	51840	1560
13	20	0.7	1.0	9	4	1015	51840	1560
14	20	0.8	1.0	9	4	1015	51840	1560
15	20	0.9	1.0	9	4	1015	51840	1560
16	20	1.1	1.0	9	4	1015	51840	1560
17	20	1.2	1.0	9	4	1015	51840	1560
18	20	1.3	1.0	9	4	1015	51840	1560
19	20	1.4	1.0	9	4	1015	51840	1560
20	20	1.5	1.0	9	4	1015	51840	1560
21	30	1.0	0.5	9	4	1525	51840	2340
22	30	1.0	0.6	9	4	1525	51840	2340
23	30	1.0	0.7	9	4	1525	51840	2340
24	30	1.0	0.8	9	4	1525	51840	2340
25	30	1.0	0.9	9	4	1525	51840	2340
26	30	1.0	1.0	9	4	1525	51840	2340
27	30	1.0	1.1	9	4	1525	51840	2340
28	30	1.0	1.2	9	4	1525	51840	2340
29	30	1.0	1.3	9	4	1525	51840	2340
30	30	1.0	1.4	9	4	1525	51840	2340
31	30	1.0	1.5	9	4	1525	51840	2340
32	30	0.5	1.0	9	4	1525	51840	2340
33	30	0.6	1.0	9	4	1525	51840	2340
34	30	0.7	1.0	9	4	1525	51840	2340
35	30	0.8	1.0	9	4	1525	51840	2340
36	30	0.9	1.0	9	4	1525	51840	2340
37	30	1.1	1.0	9	4	1525	51840	2340
38	30	1.2	1.0	9	4	1525	51840	2340
39	30	1.3	1.0	9	4	1525	51840	2340
40	30	1.4	1.0	9	4	1525	51840	2340
41	30	1.5	1.0	9	4	1525	51840	2340
42	50	1.0	0.5	9	4	2545	51840	3900
43	50	1.0	0.6	9	4	2545	51840	3900
44	50	1.0	0.7	9	4	2545	51840	3900
45	50	1.0	0.8	9	4	2545	51840	3900
46	50	1.0	0.9	9	4	2545	51840	3900
47	50	1.0	1.0	9	4	2545	51840	3900
48	50	1.0	1.1	9	4	2545	51840	3900
49	50	1.0	1.2	9	4	2545	51840	3900
50	50	1.0	1.3	9	4	2545	51840	3900
51	50	1.0	1.4	9	4	2545	51840	3900
52	50	1.0	1.5	9	4	2545	51840	3900
53	50	0.5	1.0	9	4	2545	51840	3900
54	50	0.6	1.0	9	4	2545	51840	3900
55	50	0.7	1.0	9	4	2545	51840	3900
56	50	0.8	1.0	9	4	2545	51840	3900
57	50	0.9	1.0	9	4	2545	51840	3900
58	50	1.1	1.0	9	4	2545	51840	3900
59	50	1.2	1.0	9	4	2545	51840	3900
60	50	1.3	1.0	9	4	2545	51840	3900
61	50	1.4	1.0	9	4	2545	51840	3900
62	50	1.5	1.0	9	4	2545	51840	3900
63	150	1.0	0.5	9	4	7645	51840	11700
64	150	1.0	0.6	9	4	7645	51840	11700
65	150	1.0	0.7	9	4	7645	51840	11700
66	150	1.0	0.8	9	4	7645	51840	11700
67	150	1.0	0.9	9	4	7645	51840	11700
68	150	1.0	1.0	9	4	7645	51840	11700
69	150	1.0	1.1	9	4	7645	51840	11700
70	150	1.0	1.2	9	4	7645	51840	11700
71	150	1.0	1.3	9	4	7645	51840	11700
72	150	1.0	1.4	9	4	7645	51840	11700
73	150	1.0	1.5	9	4	7645	51840	11700
74	150	0.5	1.0	9	4	7645	51840	11700
75	150	0.6	1.0	9	4	7645	51840	11700
76	150	0.7	1.0	9	4	7645	51840	11700
77	150	0.8	1.0	9	4	7645	51840	11700
78	150	0.9	1.0	9	4	7645	51840	11700
79	150	1.1	1.0	9	4	7645	51840	11700
80	150	1.2	1.0	9	4	7645	51840	11700
81	150	1.3	1.0	9	4	7645	51840	11700
82	150	1.4	1.0	9	4	7645	51840	11700
83	150	1.5	1.0	9	4	7645	51840	11700

(a) *Trials. The* `cost multiplier` *column multiplies all costs in the matrices A, B, and C (*setup, switching, *and* per-individual *costs) by the same factor. An entry μ in the* `effect multiplier` *column alters the intervention effectiveness as $P_{ijt} \mapsto 1 - \mu \cdot (1 - P_{ijt})$ for all i, j, t.*

Trial	no policy obj	solver obj	lagr heuristic obj	lagr LB	solver LB	solver vs lagr lb gap	lagr vs lagr lb gap
0	981337	911998	917859	848214	419913	0.075198	0.082108
1	981337	849522	853494	783849	331882	0.083782	0.088850
2	981337	765761	769378	699733	264131	0.094361	0.099531
3	981337	663172	666750	638446	212669	0.038729	0.044334
4	981337	549618	553150	517439	173938	0.062189	0.069016
5	981337	437759	441260	431629	144583	0.014203	0.022314
6	981337	339740	343237	332324	122136	0.022316	0.032840
7	981337	261968	265487	251332	104626	0.042318	0.056319
8	981337	204579	208130	188326	92231	0.086301	0.105157
9	981337	164092	164134	152041	83723	0.079261	0.079534
10	981337	136143	136143	122793	75635	0.108722	0.108722
11	981337	404767	406435	371615	122238	0.089210	0.093699
12	981337	411365	413399	403768	126955	0.018814	0.023853
13	981337	417963	420364	410733	131510	0.017603	0.023449
14	981337	424561	427329	417698	135953	0.016432	0.023058
15	981337	431160	434294	424663	140321	0.015299	0.022680
16	981337	444358	448225	438594	148717	0.013142	0.021959
17	981337	450957	455191	445560	152780	0.012114	0.021616
18	981337	457557	462157	416504	157077	0.098565	0.109610
19	981337	464157	469123	423470	160763	0.096079	0.107807
20	981337	470757	476090	430437	164937	0.093672	0.106063
21	1003006	1002811	1003006	956717	442992	0.048179	0.048383
22	1003006	997719	1003006	936572	350246	0.065288	0.070934
23	1003006	986950	1003006	909866	278578	0.084720	0.102367
24	1003006	969031	990655	956992	224071	0.012579	0.035176
25	1003006	924637	931146	915596	268121	0.009875	0.016984
26	1003006	859026	863490	848159	216261	0.012813	0.018076
27	1003006	768259	771990	754275	178042	0.018539	0.023486
28	1003006	652387	655975	634342	149849	0.028447	0.034102
29	1003006	522118	525676	497912	128241	0.048614	0.055760
30	1003006	398051	401603	364386	83034	0.092388	0.102134
31	1003006	298470	302041	293519	73432	0.016866	0.029033
32	1003006	808487	810340	757195	188215	0.067739	0.070187
33	1003006	818708	820970	757195	194086	0.081238	0.084225
34	1003006	828903	831600	757195	199769	0.094701	0.098263
35	1003006	839019	842230	826899	205360	0.014658	0.018541
36	1003006	849068	852860	837529	210905	0.013777	0.018305
37	1003006	868926	874121	858790	221571	0.011803	0.017852
38	1003006	878812	884752	869421	225490	0.010802	0.017634
39	1003006	888698	895383	880052	230292	0.009825	0.017421
40	1003006	898584	906014	890683	235024	0.008871	0.017213
41	1003006	908471	916645	901314	239745	0.007940	0.017010
42	1003570	1003408	1003570	961654	444081	0.043419	0.043588
43	1003570	999900	1003570	945001	351140	0.058094	0.061978
44	1003570	991941	1003570	924525	279286	0.072919	0.085499
45	1003570	980161	983218	968056	224625	0.012504	0.015662
46	1003570	965127	973238	911848	268956	0.058429	0.067324
47	1003570	946180	964433	870732	216591	0.086649	0.107612
48	1003570	922615	985101	902827	223641	0.021918	0.091130
49	1003570	890870	943840	856794	183296	0.039772	0.101595
50	1003570	843094	852618	801300	197689	0.052158	0.064044
51	1003570	761036	766913	735296	173693	0.035007	0.042999
52	1003570	649849	654125	633927	138790	0.025116	0.031861
53	1003570	901317	923735	833940	188454	0.080793	0.107676
54	1003570	911541	948232	907164	194336	0.004825	0.045271
55	1003570	921155	946571	915804	200036	0.005843	0.033596
56	1003570	930129	954897	921255	205640	0.009632	0.036517
57	1003570	938381	956741	926190	211217	0.013162	0.032986
58	1003570	953493	968058	875090	221926	0.089594	0.106239
59	1003570	960221	971691	882291	225047	0.088326	0.101327
60	1003570	966376	974670	898208	230199	0.075893	0.085127
61	1003570	972189	979469	926299	235178	0.049541	0.057401
62	1003570	977660	982747	948020	239964	0.031265	0.036631
63	1003571	1003408	1003571	961662	444042	0.043410	0.043579
64	1003571	999904	1003571	945021	351106	0.058076	0.061956
65	1003571	991956	1003571	924574	279257	0.072879	0.085442
66	1003571	980221	1003571	947565	224601	0.034463	0.059105
67	1003571	965399	1003571	928254	266957	0.040016	0.081138
68	1003571	947445	1003571	917139	214692	0.033044	0.094240
69	1003571	927570	933491	903742	223532	0.026365	0.032917
70	1003571	908331	914037	890336	183102	0.020212	0.026620
71	1003571	892903	908859	833800	197042	0.070883	0.090020
72	1003571	881521	922966	871484	186224	0.011518	0.059074
73	1003571	872333	876923	858490	181237	0.016124	0.021471
74	1003571	903823	917708	883151	187539	0.023407	0.039129
75	1003571	913845	927963	897455	194279	0.018263	0.033994
76	1003571	923233	924878	909099	199987	0.015547	0.017357
77	1003571	931811	935148	913497	204117	0.020048	0.023701
78	1003571	939832	1003571	917061	209486	0.024830	0.094334
79	1003571	954476	1003571	917139	219830	0.040710	0.094240
80	1003571	960955	1003571	917139	227144	0.047775	0.094240
81	1003571	966938	1003571	917061	230161	0.054388	0.094334
82	1003571	972683	1003571	917139	235138	0.060562	0.094240
83	1003571	977980	1003571	917139	239925	0.066338	0.094240

(b) *Accuracy results.* “lagr des L1 optGap*” : 0.03;* “lagr des L2 optGap*” : 0.10;* “lagr des optGap*” : 0.10;* “solver des optGap*” : 0.80;* “solver optGap*” : 0.80;* “lagr optGap*” : 0.00.*

Trial	solver timeToSolve	lagr timeToSolve	lagr timeToSolve L1 total	lagr timeToSolve L2 total
0	2.482967	5	1	1
1	2.096969	5	1	1
2	2.079341	5	1	1
3	2.220262	10	3	3
4	1.960423	9	2	3
5	1.965445	14	3	5
6	2.166659	14	4	6
7	2.022381	17	3	8
8	2.119086	16	3	8
9	2.179632	26	5	15
10	2.107720	48	8	30
11	2.121042	6	1	2
12	1.991170	13	3	5
13	2.041564	14	3	5
14	2.115897	14	4	5
15	2.127398	14	3	6
16	2.244456	13	3	5
17	2.033282	14	4	5
18	2.016206	10	2	3
19	2.142517	10	2	4
20	2.087467	10	2	3
21	3.498749	3	0	0
22	3.751435	3	0	0
23	3.522416	3	0	1
24	3.726861	18	3	9
25	4.910049	25	3	16
26	4.752062	31	4	22
27	4.690372	27	3	18
28	5.023750	30	3	21
29	4.959678	33	3	24
30	3.700910	67	3	58
31	3.406219	84	5	71
32	4.456458	12	2	6
33	4.606007	12	2	6
34	4.892262	12	2	6
35	5.236483	30	3	21
36	4.790893	30	3	21
37	4.967185	30	3	21
38	4.755600	31	3	21
39	5.028995	31	4	21
40	5.026888	31	4	22
41	5.119310	31	4	22
42	8.489126	5	1	1
43	8.681596	5	0	1
44	9.843386	6	0	1
45	9.423632	31	7	13
46	13.455211	50	6	33
47	12.173909	64	6	48
48	18.682212	99	7	82
49	17.542033	286	7	268
50	25.468293	296	7	279
51	25.876694	425	7	408
52	26.552296	436	7	418
53	14.562665	45	4	35
54	14.093223	73	7	56
55	12.743322	73	7	56
56	13.199386	75	7	57
57	13.101548	81	7	63
58	12.627672	64	6	48
59	16.143134	72	7	55
60	12.486596	65	7	47
61	13.343809	78	7	61
62	12.587170	85	7	67
63	61.167485	25	1	3
64	70.142685	28	1	5
65	70.277099	27	1	4
66	67.626270	91	22	27
67	107.885441	101	21	38
68	122.285184	134	23	68
69	108.818467	402	45	295
70	123.477619	522	45	416
71	210.500806	545	45	437
72	197.547928	690	63	541
73	259.837540	913	81	727
74	107.421505	195	32	100
75	102.208016	241	48	130
76	105.135661	218	45	109
77	103.143571	266	49	155
78	95.079880	153	21	89
79	111.768125	137	21	74
80	121.476973	213	22	148
81	91.711890	145	21	81
82	92.300639	139	22	74
83	96.907888	223	22	157

(c) *Timing results*

Trial	no policy cost	no policy deaths	solver cost	solver policy cost	solver disease cost	solver deaths	lagr cost	lagr policy cost	lagr disease cost	lagr deaths
0	9.81e11	9.72e04	9.12e11	6.15e10	8.51e11	8.42e04	9.18e11	6.96e10	8.48e11	8.40e04
1	9.81e11	9.72e04	8.50e11	6.39e10	7.86e11	7.78e04	8.53e11	6.96e10	7.84e11	7.76e04
2	9.81e11	9.72e04	7.66e11	6.60e10	7.00e11	6.93e04	7.69e11	6.96e10	7.00e11	6.93e04
3	9.81e11	9.72e04	6.63e11	6.60e10	5.97e11	5.91e04	6.67e11	6.96e10	5.97e11	5.91e04
4	9.81e11	9.72e04	5.50e11	6.60e10	4.84e11	4.78e04	5.53e11	6.96e10	4.84e11	4.78e04
5	9.81e11	9.72e04	4.38e11	6.60e10	3.72e11	3.68e04	4.41e11	6.96e10	3.72e11	3.68e04
6	9.81e11	9.72e04	3.40e11	6.60e10	2.74e11	2.71e04	3.43e11	6.96e10	2.74e11	2.71e04
7	9.81e11	9.72e04	2.62e11	6.60e10	1.96e11	1.94e04	2.65e11	6.96e10	1.96e11	1.94e04
8	9.81e11	9.72e04	2.05e11	6.60e10	1.39e11	1.37e04	2.08e11	6.96e10	1.38e11	1.37e04
9	9.81e11	9.72e04	1.64e11	6.56e10	9.85e10	9.74e03	1.64e11	6.60e10	9.82e10	9.70e03
10	9.81e11	9.72e04	1.36e11	6.56e10	7.06e10	6.98e03	1.36e11	6.56e10	7.06e10	6.98e03
11	9.81e11	9.72e04	4.05e11	3.30e10	3.72e11	3.68e04	4.06e11	3.48e10	3.72e11	3.68e04
12	9.81e11	9.72e04	4.11e11	3.96e10	3.72e11	3.68e04	4.13e11	4.18e10	3.72e11	3.68e04
13	9.81e11	9.72e04	4.18e11	4.62e10	3.72e11	3.68e04	4.20e11	4.87e10	3.72e11	3.68e04
14	9.81e11	9.72e04	4.25e11	5.28e10	3.72e11	3.68e04	4.27e11	5.57e10	3.72e11	3.68e04
15	9.81e11	9.72e04	4.31e11	5.94e10	3.72e11	3.68e04	4.34e11	6.27e10	3.72e11	3.68e04
16	9.81e11	9.72e04	4.44e11	7.26e10	3.72e11	3.68e04	4.48e11	7.66e10	3.72e11	3.68e04
17	9.81e11	9.72e04	4.51e11	7.92e10	3.72e11	3.68e04	4.55e11	8.36e10	3.72e11	3.68e04
18	9.81e11	9.72e04	4.58e11	8.58e10	3.72e11	3.68e04	4.62e11	9.05e10	3.72e11	3.68e04
19	9.81e11	9.72e04	4.64e11	9.24e10	3.72e11	3.68e04	4.69e11	9.75e10	3.72e11	3.68e04
20	9.81e11	9.72e04	4.71e11	9.90e10	3.72e11	3.68e04	4.76e11	1.04e11	3.72e11	3.68e04
21	1.00e12	9.94e04	1.00e12	9.00e08	1.00e12	9.92e04	1.00e12	0.00e00	1.00e12	9.94e04
22	1.00e12	9.94e04	9.98e11	2.74e10	9.70e11	9.61e04	1.00e12	0.00e00	1.00e12	9.94e04
23	1.00e12	9.94e04	9.87e11	4.24e10	9.45e11	9.36e04	1.00e12	0.00e00	1.00e12	9.94e04
24	1.00e12	9.94e04	9.69e11	6.39e10	9.05e11	8.97e04	9.91e11	9.41e10	8.97e11	8.88e04
25	1.00e12	9.94e04	9.25e11	9.60e10	8.29e11	8.21e04	9.31e11	1.06e11	8.25e11	8.17e04
26	1.00e12	9.94e04	8.59e11	9.93e10	7.60e11	7.52e04	8.63e11	1.06e11	7.57e11	7.50e04
27	1.00e12	9.94e04	7.68e11	1.02e11	6.66e11	6.60e04	7.72e11	1.06e11	6.66e11	6.59e04
28	1.00e12	9.94e04	6.52e11	1.03e11	5.50e11	5.44e04	6.56e11	1.06e11	5.50e11	5.44e04
29	1.00e12	9.94e04	5.22e11	1.03e11	4.19e11	4.15e04	5.26e11	1.06e11	4.19e11	4.15e04
30	1.00e12	9.94e04	3.98e11	1.03e11	2.95e11	2.92e04	4.02e11	1.06e11	2.95e11	2.92e04
31	1.00e12	9.94e04	2.98e11	1.03e11	1.96e11	1.94e04	3.02e11	1.06e11	1.96e11	1.94e04
32	1.00e12	9.94e04	8.08e11	5.11e10	7.57e11	7.50e04	8.10e11	5.31e10	7.57e11	7.50e04
33	1.00e12	9.94e04	8.19e11	6.13e10	7.57e11	7.50e04	8.21e11	6.38e10	7.57e11	7.50e04
34	1.00e12	9.94e04	8.29e11	7.10e10	7.58e11	7.51e04	8.32e11	7.44e10	7.57e11	7.50e04
35	1.00e12	9.94e04	8.39e11	8.08e10	7.58e11	7.51e04	8.42e11	8.50e10	7.57e11	7.50e04
36	1.00e12	9.94e04	8.49e11	9.01e10	7.59e11	7.52e04	8.53e11	9.57e10	7.57e11	7.50e04
37	1.00e12	9.94e04	8.69e11	1.09e11	7.60e11	7.53e04	8.74e11	1.17e11	7.57e11	7.50e04
38	1.00e12	9.94e04	8.79e11	1.19e11	7.60e11	7.53e04	8.85e11	1.28e11	7.57e11	7.50e04
39	1.00e12	9.94e04	8.89e11	1.29e11	7.60e11	7.53e04	8.95e11	1.38e11	7.57e11	7.50e04
40	1.00e12	9.94e04	8.99e11	1.38e11	7.60e11	7.53e04	9.06e11	1.49e11	7.57e11	7.50e04
41	1.00e12	9.94e04	9.08e11	1.48e11	7.60e11	7.53e04	9.17e11	1.59e11	7.57e11	7.50e04
42	1.00e12	9.94e04	1.00e12	9.00e08	1.00e12	9.93e04	1.00e12	0.00e00	1.00e12	9.94e04
43	1.00e12	9.94e04	1.00e12	2.66e10	9.73e11	9.64e04	1.00e12	0.00e00	1.00e12	9.94e04
44	1.00e12	9.94e04	9.92e11	3.71e10	9.55e11	9.46e04	1.00e12	0.00e00	1.00e12	9.94e04
45	1.00e12	9.94e04	9.80e11	4.80e10	9.32e11	9.23e04	9.83e11	3.90e10	9.44e11	9.35e04
46	1.00e12	9.94e04	9.65e11	6.10e10	9.04e11	8.96e04	9.73e11	3.70e10	9.36e11	9.27e04
47	1.00e12	9.94e04	9.46e11	7.72e10	8.69e11	8.61e04	9.64e11	3.62e10	9.28e11	9.19e04
48	1.00e12	9.94e04	9.23e11	9.42e10	8.28e11	8.21e04	9.85e11	1.59e11	8.26e11	8.18e04
49	1.00e12	9.94e04	8.91e11	1.19e11	7.72e11	7.65e04	9.44e11	1.59e11	7.85e11	7.77e04
50	1.00e12	9.94e04	8.43e11	1.63e11	6.80e11	6.74e04	8.53e11	1.80e11	6.73e11	6.67e04
51	1.00e12	9.94e04	7.61e11	1.71e11	5.90e11	5.85e04	7.67e11	1.80e11	5.87e11	5.82e04
52	1.00e12	9.94e04	6.50e11	1.73e11	4.76e11	4.72e04	6.54e11	1.80e11	4.75e11	4.70e04
53	1.00e12	9.94e04	9.01e11	5.35e10	8.48e11	8.40e04	9.24e11	8.98e10	8.34e11	8.26e04
54	1.00e12	9.94e04	9.12e11	5.85e10	8.53e11	8.45e04	9.48e11	9.82e10	8.50e11	8.42e04
55	1.00e12	9.94e04	9.21e11	6.58e10	8.55e11	8.47e04	9.47e11	3.83e10	9.08e11	9.00e04
56	1.00e12	9.94e04	9.30e11	6.92e10	8.61e11	8.53e04	9.55e11	4.17e10	9.13e11	9.05e04
57	1.00e12	9.94e04	9.38e11	7.09e10	8.67e11	8.59e04	9.57e11	3.62e10	9.21e11	9.12e04
58	1.00e12	9.94e04	9.53e11	7.69e10	8.77e11	8.68e04	9.68e11	3.99e10	9.28e11	9.19e04
59	1.00e12	9.94e04	9.60e11	7.61e10	8.84e11	8.76e04	9.72e11	4.30e10	9.29e11	9.20e04
60	1.00e12	9.94e04	9.66e11	7.66e10	8.90e11	8.81e04	9.75e11	4.70e10	9.28e11	9.19e04
61	1.00e12	9.94e04	9.72e11	7.95e10	8.93e11	8.84e04	9.79e11	4.59e10	9.34e11	9.25e04
62	1.00e12	9.94e04	9.78e11	7.97e10	8.98e11	8.89e04	9.83e11	4.91e10	9.34e11	9.25e04
63	1.00e12	9.94e04	1.00e12	9.00e08	1.00e12	9.93e04	1.00e12	0.00e00	1.00e12	9.94e04
64	1.00e12	9.94e04	1.00e12	2.66e10	9.73e11	9.64e04	1.00e12	0.00e00	1.00e12	9.94e04
65	1.00e12	9.94e04	9.92e11	3.71e10	9.55e11	9.46e04	1.00e12	0.00e00	1.00e12	9.94e04
66	1.00e12	9.94e04	9.80e11	4.80e10	9.32e11	9.23e04	1.00e12	0.00e00	1.00e12	9.94e04
67	1.00e12	9.94e04	9.65e11	5.94e10	9.06e11	8.97e04	1.00e12	0.00e00	1.00e12	9.94e04
68	1.00e12	9.94e04	9.47e11	7.27e10	8.75e11	8.66e04	1.00e12	0.00e00	1.00e12	9.94e04
69	1.00e12	9.94e04	9.28e11	8.60e10	8.42e11	8.34e04	9.33e11	1.09e11	8.24e11	8.16e04
70	1.00e12	9.94e04	9.08e11	9.44e10	8.14e11	8.06e04	9.14e11	1.17e11	7.97e11	7.90e04
71	1.00e12	9.94e04	8.93e11	9.65e10	7.96e11	7.89e04	9.09e11	6.40e10	8.45e11	8.37e04
72	1.00e12	9.94e04	8.82e11	9.45e10	7.87e11	7.80e04	9.23e11	1.33e11	7.89e11	7.82e04
73	1.00e12	9.94e04	8.72e11	9.18e10	7.81e11	7.73e04	8.77e11	8.55e10	7.91e11	7.84e04
74	1.00e12	9.94e04	9.04e11	5.18e10	8.52e11	8.44e04	9.18e11	4.07e10	8.77e11	8.69e04
75	1.00e12	9.94e04	9.14e11	5.83e10	8.56e11	8.48e04	9.28e11	4.76e10	8.80e11	8.72e04
76	1.00e12	9.94e04	9.23e11	6.34e10	8.60e11	8.52e04	9.25e11	7.18e10	8.53e11	8.45e04
77	1.00e12	9.94e04	9.32e11	6.63e10	8.66e11	8.57e04	9.35e11	8.18e10	8.53e11	8.45e04
78	1.00e12	9.94e04	9.40e11	7.09e10	8.69e11	8.61e04	1.00e12	0.00e00	1.00e12	9.94e04
79	1.00e12	9.94e04	9.54e11	7.47e10	8.80e11	8.71e04	1.00e12	0.00e00	1.00e12	9.94e04
80	1.00e12	9.94e04	9.61e11	7.41e10	8.87e11	8.79e04	1.00e12	0.00e00	1.00e12	9.94e04
81	1.00e12	9.94e04	9.67e11	7.66e10	8.90e11	8.82e04	1.00e12	0.00e00	1.00e12	9.94e04
82	1.00e12	9.94e04	9.73e11	7.73e10	8.95e11	8.87e04	1.00e12	0.00e00	1.00e12	9.94e04
83	1.00e12	9.94e04	9.78e11	7.69e10	9.01e11	8.93e04	1.00e12	0.00e00	1.00e12	9.94e04

(d) *System outcome results*

3.2.1 Summaries of Results

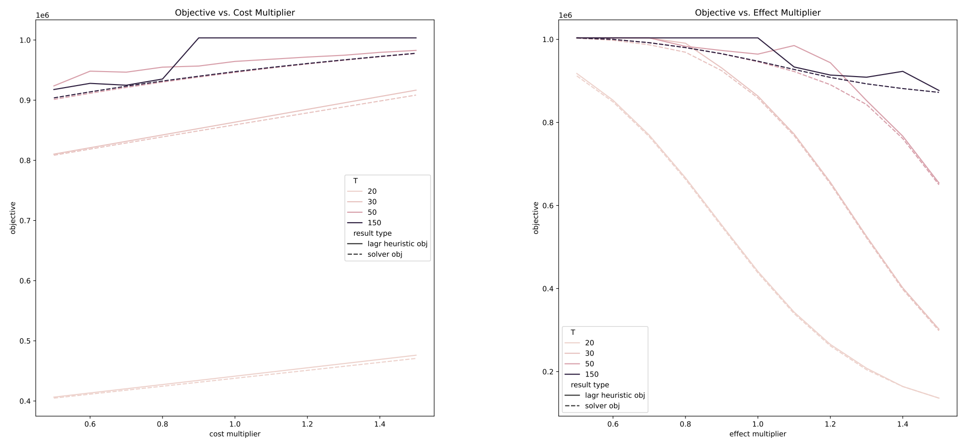


Figure 5: *Objective value vs. factor by which policy costs are all changed; objective vs. factor by which policy effectivenesses are all changed.*

3.3 Lagrangian Subproblem Quasiconvexity

The subproblem $L2(\lambda)$ defined in Section 2.1 has no integer constraints, but it still takes considerable time to solve with the BARON numerical solver. If the problem has suitable structure, such as quasiconvexity, a basic gradient descent algorithm might suffice for part of its solution. In particular, we examined whether $L2(\lambda)$ is quasiconvex in $P_t, t = 1, \ldots, T$.

A function $f(\boldsymbol{x})$ is quasiconvex if and only if for any two values $\boldsymbol{x}_1$, and $\boldsymbol{x}_2$ in its domain, the function of one variable $V(\theta) = f((1-\theta)\boldsymbol{x}_1 + \theta\boldsymbol{x}_2)$ is quasiconvex for values of $\theta \in [0,1]$.

To probe this necessary and sufficient condition for quasiconvexity in P, we define the function $V_{P^1,P^2} : [0,1] \to \mathbb{R}$ for any values P^1, P^2 where P_t^1 and P_t^2 are fixed for all $t = 1, \ldots, T$. This function is such that $V_{P^1,P^2}(\theta)$ is the optimal value of $L2(\lambda)$ where $P_t = (1-\theta)P_t^1 + \theta P_t^2$ for all $t = 1, \ldots, T$.

The following plots illustrate the value of $V_{P^1,P^2}(\theta)$ for $\theta \in [0,1]$. If the curves appear quasiconvex, then a necessary condition is met for $L2(\lambda)$ being quasiconvex in P. In each trial, the values $P_t^1, P_t^2, t = 1, \ldots, T$ were generated in the interval $[0,1]$ uniformly randomly[4].

[4] In fact, for this model P_t cannot be equal to 0, because of its definition in the full model as a product of "intervention effectiveness" factors. So, the lowest value it can possibly take on is the product of the effectiveness factors of *all* possible interventions. Both in the full model and in this investigation of quasiconvexity, values of P_t are constrained to be in the interval $[P_{lb}, 1]$, where P_{lb} is this small value. Since the logarithm of P_t is part of the model, the domain becomes effectively open (the objective is undefined at 0), this adjustment improves performance of the solver by giving a closed domain without loss of generality of solutions. The restriction of the domain for this quasiconvexity test is also without loss of generality, because for the purposes of this model, only values of P_t in the constrained interval are relevant.

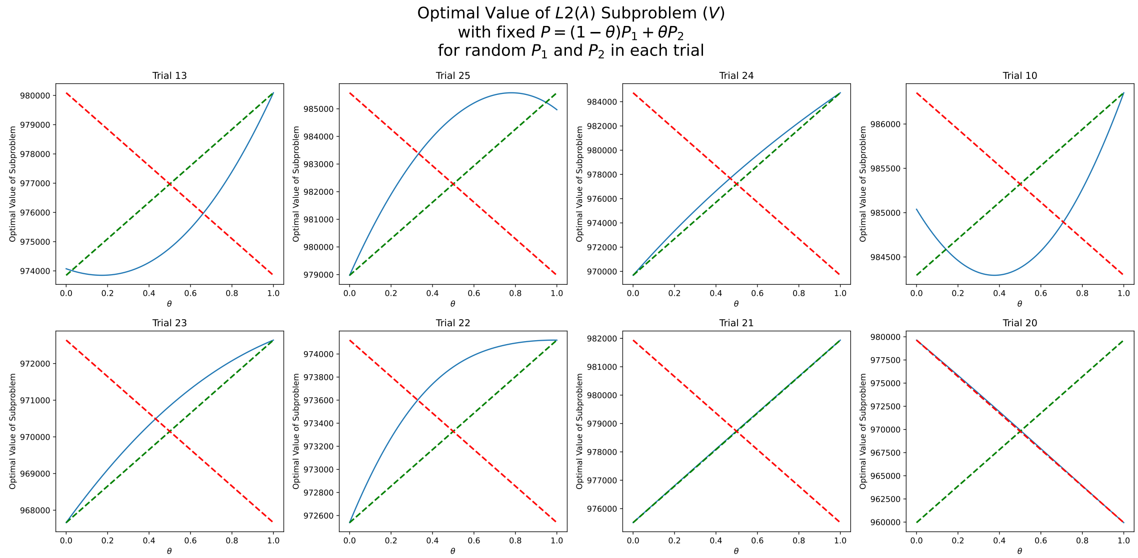


Figure 6: *Optimal value of $L2(\lambda)$ with P fixed to values that vary along a line; that is, $V_{P^1,P^2}(\theta)$ vs θ for several randomly-selected values of P^1 and P^2.*

As can be seen in Figure 6, the function V appears to be neither quasiconvex nor quasiconcave.

To probe the possibility that the function V is not jointly quasiconvex in the P_t variables, but is componentwise quasiconvex in each $P_t, t = 1, \ldots, P$, we can investigate whether the function $V_{P^1,P^2}(\theta)$ is quasiconvex for any P^1, P^2 such that the line segment connecting P^1 and P^2 is parallel to an axis P_t (for some t), i.e. by repeating the same experiment but only varying one P_t at a time.

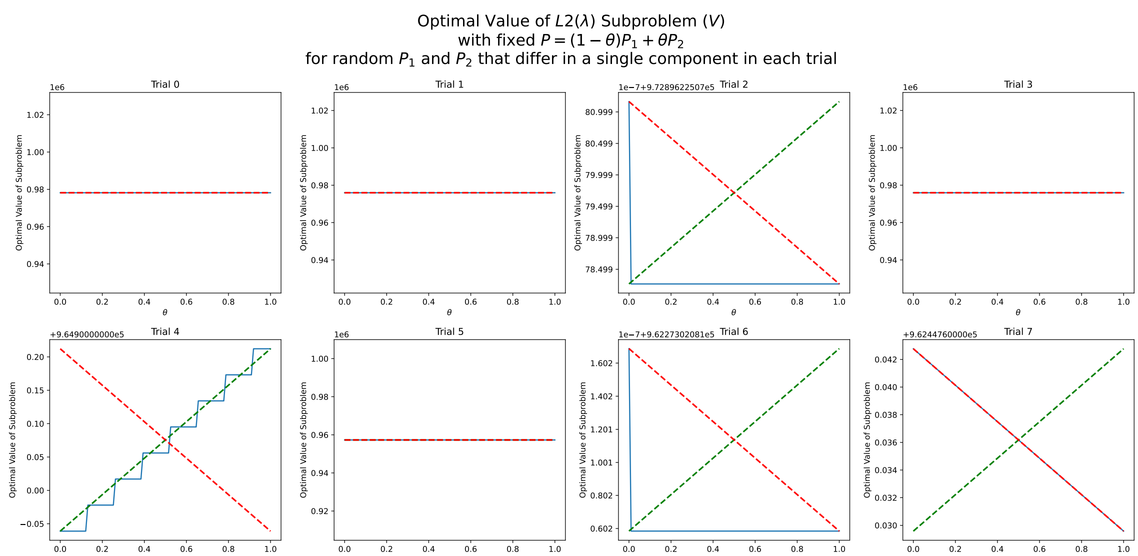


Figure 7: *The same experiment was repeated as is illustrated in Figure 6, but by only varying a single P_t component at a time.*

Based on the plots in Figure 7, it is inconclusive whether V is componentwise quasiconcave or quasiconvex. The variation in the objective of the $L2(\lambda)$ subproblem may be small with respect to any one component P_t.

4 Conclusion

References