# Worker Safety and Labor Supply Under Dangerous Working Conditions

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**Abstract**

*During a medical crisis, demand for labor from medical personnel is high, and yet workers' concern for their own safety may prevent them from working. The COVID-19 pandemic has highlighted practical and ethical challenges around allocation of scarce resources in healthcare, particularly personal protective equipment (PPE) and ventilators.*

*We consider a medical facility's problem of paying for an adequate safety level, in addition to wages, to attract risk-averse workers. We consider a population of workers whose risk aversion attenuates their response to both wages and injury.*

## 1 Introduction

Rationing of medical resources in a pandemic has immediate and grave effects. Natural questions arise, such as "Who gets the Ventilator?", the subject of the Freakonomics podcast in [10]. Several different ethical concepts are reviewed in [11], namely "maximizing the benefits produced by scarce resources, treating people equally, promoting and rewarding instrumental value, and giving priority to the worst off." They conclude that, "the proposals for allocation discussed above also recognize that all these ethical values and ways to operationalize them are compelling. No single value is sufficient alone to determine which patients should receive scarce resources. Hence, fair allocation requires a multivalue ethical framework".

One ethical concept that arises in [18], [23], [11], and [7] is considering a patient's "instrumental value," or ability to contribute to the medical effort overall. In particular, [18] draws a distinction between this *forward-looking* justification for prioritizing the care of frontline health workers and the similar, *backward-looking* concern for "reciprocity" in prioritizing those same workers (distinguished as *retrospective* and *prospective* concerns in [11]). They note that, while medical ethicists have promoted prioritization of resources for health workers (particularly in [11]), some US states write these concerns into policy (such as Michigan) and others don't (such as New York and Minnesota).

Allocation of medical resources for frontline medical workers has garnered the focus of the works cited above, and emotionally-charged support for medical personnel is ubiquitous in the media. As their prospective instrumental value motivates support for medical personnel among the public and in policy, it is worth examining the extent to which that support, in the form of investment in worker safety and compensation overall, affects the medical labor supply.

This work considers the effect of investment in workplace safety on medical workers' willingness to subject themselves to dangerous working conditions. In particular, with a low wage or safety level, it is not in workers' interest to work; the medical facility attempts to attract labor by setting a wage and a safety level to maximize its profit.

In Section 3, a model is proposed that incorporates a risk-averse population of workers, and the firm profit-maximization problem is analyzed. In Section 4, the effects of the various system parameters on optimal outcomes is explored via numerical experiments.

## 2 Literature Review

Allocation of medical resources for medical personnel to foster or increase their prospective instrumental value fits into the literature on the effect of hazard pay and workers' compensation on absenteeism. Supporting a frontline medical worker's health to extract their instrumental value in fighting a crisis is analogous to extracting value from a worker, and keeping them on the job, in normal economic conditions.

This review hasn't encountered ethical support for allocation of medical resources to medical personnel that *only* justifies this support on reciprocity and *does not* mention instrumental value (indeed [11] does not mention reciprocity at all). So, in this work, instrumental value will stand as the *only* justification for allocating resources specifically for medical personnel. That is, this work will consider the effect of fostering workplace safety on a firm's productive output, not as an end in itself.

Retaining labor supply levels during a crisis is a problem faced by medical facilities in normal times. Still, the recent COVID-19 crisis has highlighted the concern medical workers have for their own safety. An April 2020 survey by the nursing association HOLLIBLU and the data-science firm Feedtrail [2] found that 61% of respondents "said they are likely to leave their current position or specialty", while a follow-up in May 2020 [4] found that 46% reported the same. National Nurses United also conducted a survey in May 2020 [3], and found that 87% of respondeds reported "having to reuse a single-use disposable respirator or mask with a COVID-19 patient." Both these surveys were based on convenience samples, but the seeming frustration with increasingly unsafe working conditions is striking.

Both [19] and [18] note differences in state policy for allocating scarce medical supplies. In particular, New York State policy explicitly tries to avoid all resources being allocated to one particular group, one being medical personnel. Conversely, it is also important that no group be entirely excluded from critical care, and [11] [18] and [23] all condemn the emergence of policies and guidelines that recommend against allocating resources for the disabled.

The construction of a "multivalue ethical framework" is the subject of [18], which analyzes the overall utility of a matching policy. Their scope is that of distributing any scarce public goods, such as affirmative-action admissions and visa allocation, but they focus on allocation of scarce medical supplies, such as ventilators, PPE, vaccines, and ICU beds.

They conclude that separate "soft reserves" may have better results than a single priority queue:

> "While practical, priority point mechanisms are limiting for a number of reasons. First, priority points norm or scale different and potentially-unrelated ethical principles into one dimension. These challenges are like the usual ones associated with aggregating social alternatives into a single ordering based on multiple inputs – a situation which involves 'comparing apples to oranges."'

The separate reserves ensure both that no single group receives all resources, and that no groups are excluded; this may reduce any perception of unfairness as a result. In [18], this perception is explored by examining levels of community involvement in various other scarce allocation solutions.

The perception of unfairness may be as difficult to operationalize as the separate concerns in the first place, however. Hence this work does not focus on perception of unfairness in general, or on balancing ethical concerns in particular. Instead, the focus is on focus on forward-looking or "prospective" (rather than backward-looking or "retrospective") ethical consideration. The only such value mentioned in the literature seems to be that of instrumental value. The hope is to operationalize this concept in an economic model.

In a broad sense, allocating medical resources for medical personnel should be able to increase their productivity and combat a crisis more effectively than allocating those resources for another group.

In a review of absenteeism [6], it is noted that some forms of compensation for dangerous conditions *increase* work absence, such as higher "short-term accident and sickness benefits" for absent employees. Overall, absenteeism is framed as a cost to employers that can be avoided via compensation. They note "a family of indifference curves between wages and absences can be derived for each individual worker. Because of differences in tastes and outside income, there is a different family of such curves for each individual...Workers who prefer more absences and a lower wage...obtain jobs with employers who find absenteeism relatively inexpensive." In this paper, employees less willing to work for higher wages and less willing to subject themselves to dangerous working conditions would be considered more risk-averse. Productivity and absences (other than turnover) are not considered in this work.

In [20], they consider factors that prevent workers "shirking" via absence, quit rate, or otherwise. They explore the effects of paying more than the "going wage," the unemployment rate (via increasing penalty of unemployment when fired), monitoring cost and effectiveness, and even interest rate. Their primary focus is worker "effort level" rather than willingness to work at a given wage and safety level.

In [16], they consider a "lifetime increasing earnings profile" to prevent all forms of unwillingness to work. If a worker is not just losing current wages, but access to future increasing earnings, they are more inclined to work under any circumstances.

In [17], they examine difficulties in measuring all forms of absenteeism and shirking overall. In particular, they cite "absenteeism research as representing 'a hodgepodge of conceptually and operationally differing definitions'," which result in conflicting empirical relationships "between absenteeism and satisfaction with pay, promotions, and supervision." In contrast to [6] (which it preceeded by four years), they cite a conclusion that "the common view of absence as a painreductive response on the part of the worker to his work experience is naive, narrow, and empirically unsupportable." They empirically explore - without finding a single answer - whether withdrawal behavior such as shirking, absenteeism, and turnover, correlate positively, act as substitutes, or in general have common causes. They cite a study that "found that student nurses who left a hospital had higher absence rates than those nurses who stayed." To the extent that absence indicates a flatter income utility or a steeper response to negative workplace experience, workers with more absences may be more generally risk-averse, although this paper seems to warn against drawing any mechanistic conclusion from their empirical results. Still, this result would support the observation that more generally risk-averse individuals are less likely to agree to work under dangerous conditions at a given wage, a central idea in this current work.

In [12], they explore the difference between "efficiency wages" and "compensating payments" for preventing worker withdrawal behavior. Their model incorporates "probability of being dismissed for insufficient work effort (i.e. shirking)." We are instead concerned with labor scarcity and exogenous worker efficiency.

In [21] they consider the effect of wage on productivity of workers. They note that in "firms where net productivity is more sensitive to wages (with higher turnover costs, higher monitoring costs, or where shirking workers can do more damage) will find it desirable to pay higher wages for workers of identical characteristics." While the healthcare setting fits that description, we consider workers to be equally effective as long as they agree to work, and we consider the case of labor scarcity where attracting workers is the primary concern.

In [22], they try to answer the question "How much will a person pay to reduce the probability of his own death by a 'small' amount?" They ask, "How can it be known whether observed risk-wage relationships reflect mainly marginal costs of producing safety - the supply of job safety - rather than the demand for it?" They propose a model governing individual worker utility and firm profit, but 1) they do not propose functional forms for utility or production functions, and 2) they do not propose a worker population model governing the labor supply at a given wage and safety level. They suggest an extension in which the impact of injury on workers' lost wages is heterogeneous. We instead consider the case where workers lose *all* their wages for a period after injury, and instead worker heterogeneity is in their risk-response to both wage and injury magnitude.

# 3 Model

## 3.1 Worker Utility

Medical workers are likely to exhibit risk-aversion in their willingness to accept a wage in exchange for subjecting themselves to some probability of an accident. However, it is established in [15] that individuals tend to be risk-seeking when facing a certain loss. Furthermore, workers who are more risk-averse with respect to their income may be more disinclined than others to face an expensive accident.

As in [15], we use the term "risk-averse" broadly to refer to workers with decreasing marginal utility in their earnings, though their wage is not random except with respect to the probability of an accident.

The toll of an accident or injury is not confined to lost wages, and indeed insurance may remove the effect of lost wages. Hence, we consider a wage utility and injury disutility that are additively separable. Let

$$
\begin{aligned}
U(r) &= \text{utility of payment } r \\
V(H) &= \text{disutility of injury of magnitude } H \\
\Psi(r) &= U(r) - V(H) \\
&= \text{Utility of wage } r \text{ and} \\
&\quad \text{injury of (fixed) magnitude } H.
\end{aligned}
$$

where $U'(r), V'(H) > 0$, and $U''(r), V''(H) < 0$ reflecting risk-aversion in gains and risk-seeking in losses. Utility $\Psi(r)$ is the utility of income $r$ along with the pain and suffering of an injury, which must therefore satisfy $\Psi(r) < U(r)$. We consider $H$ to be exogenous, but explore its effect on firm decisions later in section 4.

In general, neither a higher nor lower income necessarily induces risk-aversion, and we consider that workers respond to income increases (via $U(r)$) with constant absolute risk aversion (CARA). In the domain of loss/injury (via $V(H)$), for simplicity we consider a (slightly different) CARA function as well. However, with the motivation that risk-averse workers are also more injury-averse, we designate one risk-aversion parameter $\theta > 0$ to attenuate risk response in both domains:

$$
\begin{aligned}
U(r) &= \frac{1}{\theta}(1 - e^{-\theta r}) \\
V(H) &= \theta'(1 - e^{-\frac{1}{\theta'}H}) \\
\theta' &= t\theta, t > 0.
\end{aligned}
$$

Note that as $\theta$ increases, $U(r)$ becomes flatter, reflecting risk-aversion in income. As $\theta'$ increases, $V(H)$ becomes *steeper*, reflecting less risk-seeking in losses. Allowing $\theta' = t\theta$ reflects that individuals' risk-aversion in income may accompany less risk-seeking behavior in losses. For simplicity, here we consider $t = 1$, or $\theta' = \theta$.

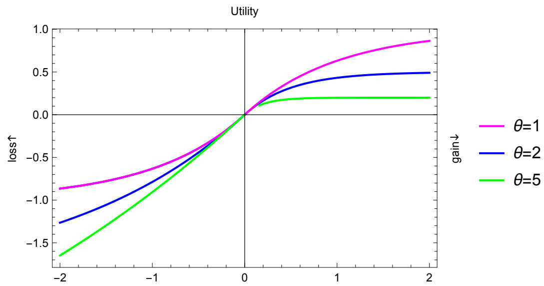


**Figure 1:** *Utility in gains and losses attenuated by a single risk-response parameter $\theta$*

Figure 1 illustrates workers' utility response to gains ($U(r)$) on the positive side of the horizontal axis, and utility response to losses ($-V(H)$) on the negative side of the horizontal axis. Note, as will be critical, that in addition to flattening and steepening the utility and disutility response curves, respectively, a higher risk-response factor $\theta$ also monotonically decreases utility at any income or injury level.

**Remark 1.** Both $U(r)$ and $-V(H)$ are decreasing in $\theta$.

*Proof.* First, $U(r)$ is decreasing in $\theta$:

$$
\begin{aligned}
\frac{d}{d\theta}\frac{1}{\theta}(1 - e^{-\theta r}) &= \frac{(1+\theta r)e^{-\theta r} - 1}{\theta^2} \\
&\leq 0 \\
&\Leftrightarrow \\
(1+\theta r)e^{-\theta r} - 1 &\leq 0 \\
&\Leftrightarrow \\
1 + \theta r &\leq e^{\theta r}
\end{aligned}
$$

which is true as $1 + x \leq e^x$ for all $x$.

In addition, $-V(H)$ is decreasing in $\theta$:

$$\begin{aligned}\frac{d}{d\theta} - \theta(1 - e^{-\frac{1}{\theta}H}) &= -1 + (1 + \frac{H}{\theta})e^{-\frac{H}{\theta}} \\ &\leq 0 \\ &\Leftrightarrow \\ 1 + \frac{H}{\theta} &\leq e^{\frac{H}{\theta}}\end{aligned}$$

which is true for the same reason. □

## 3.2 Probability of Worker Injury and Labor Supply

Let a worker normally earn wage $w$ but with probability $p$ earn no income income due to an accident or injury that causes pain and suffering $H$. We refer to $p$ as the "probability of accident/injury" and $(1-p)$ as the "safety level".

It is natural to consider a worker buying insurance at the level $I$ that maximizes their expected utility. As in [22], consider perfect, load-free insurance, which necessarily has price $\frac{p}{1-p}$ per unit insurance, so that the expected payout of insurance is 0:

$$\begin{aligned}E(\text{payout}) = &\overbrace{p(-I)}^{\text{insurer pays}} + \overbrace{(1-p)(\frac{p}{1-p}I)}^{\text{insurer is paid}} \\ &= 0.\end{aligned}$$

That is, in the case of an accident, the insurer pays $I$ to the worker; in the case of no accident, the worker pays $\frac{p}{1-p}I$. The expected worker utility is then

$$E = (1-p)U(w - \frac{p}{1-p}I) + p\Psi(I).$$

The conditions described for $U$ and $\Psi$ (concave, increasing) ensure a first-order condition will yield the optimal $I$ without substituting any functional forms:

$$\begin{aligned}0 = (1-p)U'(w - \frac{p}{1-p}I)(\frac{-p}{1-p}) + p\Psi'(I) \\ \Downarrow \\ \Psi'(I) = U'(w - \frac{p}{1-p}I).\end{aligned}$$

In the case of the CARA functional forms assumed above for $U$ and $\Psi$, this implies $I = (1-p)w$ and the worker's expected utility $E$ is

$$E = \frac{1}{\theta}(1 - e^{-\theta(1-p)w}) + p\theta(1 - e^{-\frac{H}{\theta}}).$$

We consider that a worker is willing to work at wage $w$ if expected utility $E > 0$.

## 3.3 Firm Profit

Consider a firm that hires $L$ workers at wage $w$ at accident probability level $p$. As in [22], probability of accident $p$ is endogenous, and the firm attempts to maximize profit as defined by production minus wages and cost of maintaining a desirable safety level.

The cost of maintaining safety level $(1-p)$ is $C(1-p)$, where $C(0) = 0$ and $\lim_{(1-p)\to 1} C(1-p) = \infty$. We assume $C$ to be convex and strictly increasing. A natural candidate is

$$C(1-p) = c \cdot \frac{1-p}{p}$$

for some *cost of safety* $c > 0$.

Let the firm's (monetized) *production* function $\phi(L,p)$ represent the output of the firm with $L$ employees at risk level $p$. *Production* could be represented by revenue or a monetization of services provided, such as patients served.

We consider the case of labor scarcity in which productive output may simply be proportional to labor level $L$ and independent of probability of injury $p$:

$$\phi(L,p) = \gamma L$$

where $\gamma$ represents the productive output per employee. Note that while $\phi(L,p)$ does not depend on $p$, the number of workers $L$ does depend on $p$, which assumes that a worker can be equally productive at any probability $p$ of injury such that their utility is positive. More risk-averse workers refuse to work at high $p$ (low safety), but when they work they are equally productive.

The firm's profit can then be written

$$\begin{aligned}\Pi &= \phi(L,p) - wL - C(1-p)\\ &= (\gamma - w)L - c\frac{1-p}{p}. \qquad (1)\end{aligned}$$

## 3.4 Worker Type and Population

Consider a heterogeneous population of workers characterized by their risk-response parameter $\theta \sim F(\cdot)$ where CDF $F$ has positive support. Suppose there are $\overline{L}$ total potential workers, and a fraction $\beta$ of them decide to work at a given $p$ and $w$. That is, $L$ in the firm profit function can be written

$$L = \beta \overline{L}. \qquad (2)$$

As noted above, worker utility at any wage is strictly decreasing in the risk-response parameter $\theta$: a higher $\theta$ entails a flatter and lower utility from wages, and a steeper and higher disutility from injury $H$. So, for a fixed wage $w$, probability of accident $p$, and injury level $H$, there is a risk-response $\tilde{\theta}$ above which workers will not work.

Then the fraction of workers who will work is $\beta = F(\tilde{\theta})$ where, at $\tilde{\theta}$, expected utility $E = 0$. That is, $\tilde{\theta}$ $(> 0)$ satisfies the equation

$$\begin{aligned}0 &= E\\ &= \frac{1}{\tilde{\theta}}(1 - e^{-\tilde{\theta}(1-p)w}) + p\tilde{\theta}(1 - e^{-\frac{H}{\tilde{\theta}}}).\end{aligned}$$

Unfortunately, the cutoff risk-response $\tilde{\theta}$ cannot be written analytically in terms of firm decisions $p$ and $w$. However, the wage that induces cutoff risk-response $\tilde{\theta}$ can be written

$$w = \frac{-1}{\tilde{\theta}(1-p)} \ln\left(1 - p\tilde{\theta}^2(1 - e^{-\frac{H}{\tilde{\theta}}})\right) \qquad (3)$$

where the obvious restriction that $0 < w < \infty$ corresponds to

$$p < \frac{1}{\tilde{\theta}^2(1 - e^{-\frac{H}{\tilde{\theta}}})}.$$

This is a non-convex constraint, inducing an implicit non-convex feasible region illustrated in Figure 2. Outside the shaded region, the risk and cost of an injury are sufficiently high to require an infinite wage to induce workers at risk type $\tilde{\theta}$ to work.

Finally, we assume a functional form for the worker type distribution function: $\theta \sim \text{Unif}[0, \overline{\theta}]$, and

$$\beta = F(\tilde{\theta}) = \frac{\tilde{\theta}}{\overline{\theta}}, \qquad (4)$$

where $\overline{\theta}$ is the maximum worker risk-response.

**Remark 2.** Note that the wage strictly increases in $\tilde{\theta}$, meaning that aiming to capture more risk-averse workers requires a higher wage. The wage is also strictly increasing in $p$, meaning that a more dangerous job requires a higher wage to capture equally risk-averse individuals.

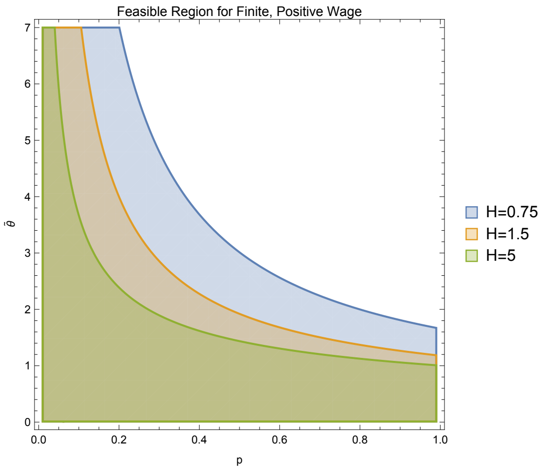


**Figure 2:** *The feasible region on probability of accident $p$ and cutoff risk-resonse $\tilde{\theta}$ resulting in a finite cutoff wage.*

*Proof.*

$$\frac{\partial w}{\partial \tilde{\theta}} = \frac{\ln(1 - p\theta^2(1 - e^{-\frac{H}{\tilde{\theta}}}))}{\tilde{\theta}} + \frac{2p\tilde{\theta}(1 - e^{-\frac{H}{\tilde{\theta}}}) - pHe^{-\frac{H}{\tilde{\theta}}}}{1 - p\tilde{\theta}^2(1 - e^{-\frac{H}{\tilde{\theta}}})} \geq 0$$

$$\Leftrightarrow$$

$$\ln(1 - p\tilde{\theta}^2(1 - e^{\frac{-H}{\tilde{\theta}}})) \geq -\frac{2p\tilde{\theta}^2(1 - e^{-\frac{H}{\tilde{\theta}}}) - pH\tilde{\theta}e^{-\frac{H}{\tilde{\theta}}}}{1 - p\tilde{\theta}^2(1 - e^{-\frac{H}{\tilde{\theta}}})}$$

The left-hand side is bounded below by:

$$\frac{-p\tilde{\theta}^2(1 - e^{-\frac{H}{\tilde{\theta}}})}{1 - p\tilde{\theta}^2(1 - e^{-\frac{H}{\tilde{\theta}}})}$$

for the simple reason that for any $x > -1$, $\ln(1 + x) \geq \frac{x}{1+x}$.

The right-hand side is bounded *above* by this same expression: they have the same denominator, and comparing their numerators reduces to $1 + \frac{H}{\tilde{\theta}} \leq e^{\frac{H}{\tilde{\theta}}}$, which is true because $1 + x \leq e^x$ for all $x$.

The proof that $\frac{\partial w}{\partial p} > 0$ is trivial. □

## 3.5 Firm Profit Maximization

Substituting the wage equation (3), the labor supply equations (4) and (2), in the firm profit (1) yields the following maximization problem for the firm:

$$\begin{aligned}
\underset{p,\tilde{\theta},w,L}{\text{Maximize}} \quad & \Pi && = (\gamma - w)L - c\frac{1-p}{p} \\
\text{s.t.} \quad & w && = \frac{-1}{\tilde{\theta}(1-p)} \ln\left(1 - p\tilde{\theta}^2(1 - e^{-\frac{H}{\tilde{\theta}}})\right) \\
& L && = \frac{\overline{L}}{\overline{\theta}}\tilde{\theta} \\
0 \leq \quad & p && \leq 1
\end{aligned}$$

where $w$ and $L$ are clearly just splitting variables. The problem can be written succinctly as:

$$\underset{p,\tilde{\theta}}{\text{Maximize}}\, \Pi = \tag{5}$$
$$\frac{\overline{L}}{\overline{\theta}}\left(\gamma\tilde{\theta} + \frac{1}{1-p}\ln\left(1 - p\tilde{\theta}^2(1 - e^{-\frac{H}{\tilde{\theta}}})\right)\right) - c\frac{1-p}{p}$$
$$0 \leq p \leq 1$$
$$0 \leq \tilde{\theta} \leq \overline{\theta}$$

**Remark 3.** Note that $\Pi$ in (5) is concave separately in both $p$ and $\tilde{\theta}$. While the (implicit) feasible region is not convex, this still permits fruitful second-order optimization methods over either variable. Furthermore, this feasible region is compact due to the bounds on $p$ and $\tilde{\theta}$, allowing straightforward sampling for numerical methods.

It would remain to show that $\Pi$ is *jointly* concave in $p$ and $\tilde{\theta}$ on its feasible set. In fact, it is not: it clearly violates Jensen's inequality (consider a secant passing over the implicitly infeasible region depicted in Figure 2, where $w \to \infty$ and $\Pi \to -\infty$). Note that the wage

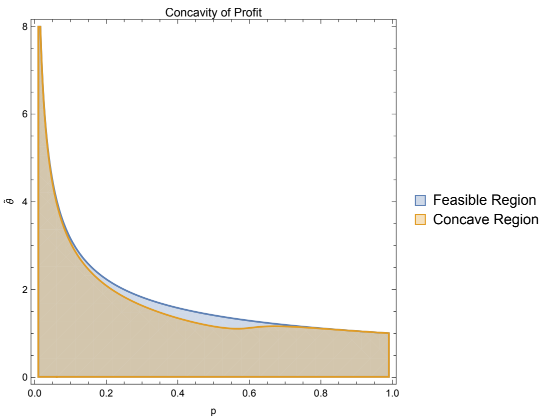


**Figure 3:** *The profit $\Pi$ seems to be concave on its entire domain.*

$$w = \frac{-1}{\tilde{\theta}(1-p)} \ln\left(1 - p\tilde{\theta}^2(1 - e^{-\frac{H}{\tilde{\theta}}})\right)$$

approaches $\infty$ when the argument of the logarithm is negative (meaning $\Pi \to -\infty$), which is when the following does not hold:

$$p < \frac{1}{\tilde{\theta}^2(1 - e^{-H/\tilde{\theta}})}. \tag{6}$$

Equation (6) defines a nonconvex region in the $(p, \tilde{\theta})$ plane on which profit is finite. Figure 3 illustrates this region, which is implicitly the domain of optimization, as the objective function is not defined outside this region.

Still, numerical experimentation seems to show that $\Pi$ is in fact jointly concave on the interior of its feasible set. A function of two variables with negative second partial derivatives is jointly concave if and only if its Hessian has positive determinant. Figure 3 also illustrates the region on which this condition holds, which seems to be the entirety of the feasible region (noting numerical instability on the boundary where $\Pi \to -\infty$).

*Proof.* ($\Pi$ concave in $p$ and $\tilde{\theta}$) Note that the second three terms (excluding $\frac{-2c}{p^3}$) of

$$\begin{aligned}\frac{\partial^2 \Pi}{\partial p^2} &= \frac{-2c}{p^3} + \frac{\overline{L}}{\tilde{\theta}} \frac{-\tilde{\theta}^4(1 - e^{-H/\tilde{\theta}})}{(1-p)(1 - p\tilde{\theta}^2(e - e^{-H/\tilde{\theta}}))^2} \\ &\quad - \frac{\overline{L}}{\tilde{\theta}} \frac{-2\tilde{\theta}^2(1 - e^{-H/\tilde{\theta}})}{(1-p)^2(1 - p\tilde{\theta}^2(e - e^{-H/\tilde{\theta}}))} \\ &\quad + \frac{\overline{L}}{\tilde{\theta}} \frac{2\ln(1 - p\tilde{\theta}^2(1 - e^{-H/\tilde{\theta}}))}{(1-p)^3}\end{aligned}$$

are negative if and only if

$$\begin{aligned}\ln(1 - p\tilde{\theta}^2(1 - e^{-H/\tilde{\theta}})) \leq{}& \frac{(1-p)\tilde{\theta}^2(1 - e^{-H/\tilde{\theta}})}{1 - p\tilde{\theta}^2(1 - e^{-H/\tilde{\theta}})} \\ &+ \frac{\frac{1}{2}(1-p)^2\tilde{\theta}^4(1 - e^{-H/\tilde{\theta}})}{1 - p\tilde{\theta}^2(1 - e^{-H/\tilde{\theta}})}.\end{aligned}$$

For all values at which the left-hand side is defined, the right-hand side is positive, and the left-hand side is negative (as $\ln(1+x) \leq x \ \forall \ x$, the left-hand side is bounded above by $-p\tilde{\theta}^2(1 - e^{-H/\tilde{\theta}}) < 0$).

Similarly, $\frac{\partial^2 \Pi}{\partial \tilde{\theta}^2} < 0$:

$$\begin{aligned}&\frac{\partial^2 \Pi}{\partial \tilde{\theta}^2} \\ &= -\frac{\overline{L}}{\tilde{\theta}} \frac{\left(Hpe^{-H/\tilde{\theta}} - \tilde{\theta}pe(1 - e^{-H/\tilde{\theta}})\right)^2}{(1-p)\left(1 - p\tilde{\theta}^2(1 - e^{-H/\tilde{\theta}})\right)^2} \\ &- \frac{\overline{L}}{\tilde{\theta}} \frac{2p(1 - e^{-H/\tilde{\theta}}) - \frac{H^2 p}{\tilde{\theta}^2} e^{-H/\tilde{\theta}} - \frac{2Hp}{\tilde{\theta}} e^{-H/\tilde{\theta}}}{(1-p)\left(1 - p\tilde{\theta}^2(1 - e^{-H/\tilde{\theta}})\right)}.\end{aligned}$$

The first term is clearly negative because of the squared terms in the numerator and denominator. The second term is negative if and only if

$$2p(1 - e^{-H/\tilde{\theta}}) \geq \left(\frac{H^2 p}{\tilde{\theta}^2} + \frac{2Hp}{\tilde{\theta}}\right) e^{-H/\tilde{\theta}}$$

which reduces to

$$e^{H/\tilde{\theta}} \geq \frac{1}{2}\left(\frac{H}{\tilde{\theta}}\right)^2 + \frac{H}{\tilde{\theta}} + 1,$$

which holds because $e^x \geq \frac{1}{2}x^2 + x + 1$ for all $x \geq 0$.

□

# 4 Sensitivity Analysis

The feasible region is compact and the profit function $\Pi$ in (5) is concave separately in $p$ and $\tilde{\theta}$ (see Remark 3). This problem lends itself to a straightforward numerical optimization method by fixing values of $p$ or $\tilde{\theta}$ and then optimizing a concave function over the other.

According to [24], Mathematica's built in optimization methods use a combination of a differential evolution algorithm and interior point methods to maximize over arbitrary regions with possibly many local optima. For simplicity, this method was employed for most plots, with MATLAB's `fmincon` function employed as well.

Note in this analysis, $\beta^*$ represents the fraction of the labor pool ($\frac{L}{\overline{L}}$) that decides to work at a profit-maximizing safety level $(1-p^*)$ and wage $w^*$. As $\theta \sim \text{Unif}[0, \overline{\theta}]$, we can write $\beta^* = \frac{\tilde{\theta}^*}{\overline{\theta}}$, which thereforealso reflects the optimal maximum risk-response level for employees.

## 4.1 $\Pi^*$ vs Injury Magnitude $H$

**Proposition 1.** *In addition to profit decreasing and necessary wage increasing, the following characterize the system response to injury magnitude $H$:*

1. *There is an injury magnitude $\underline{H}$ such that $p^*$ is decreasing in $H$ when $H < \underline{H}$, and $p^*$ is increasing in $H$ when $H > \underline{H}$*

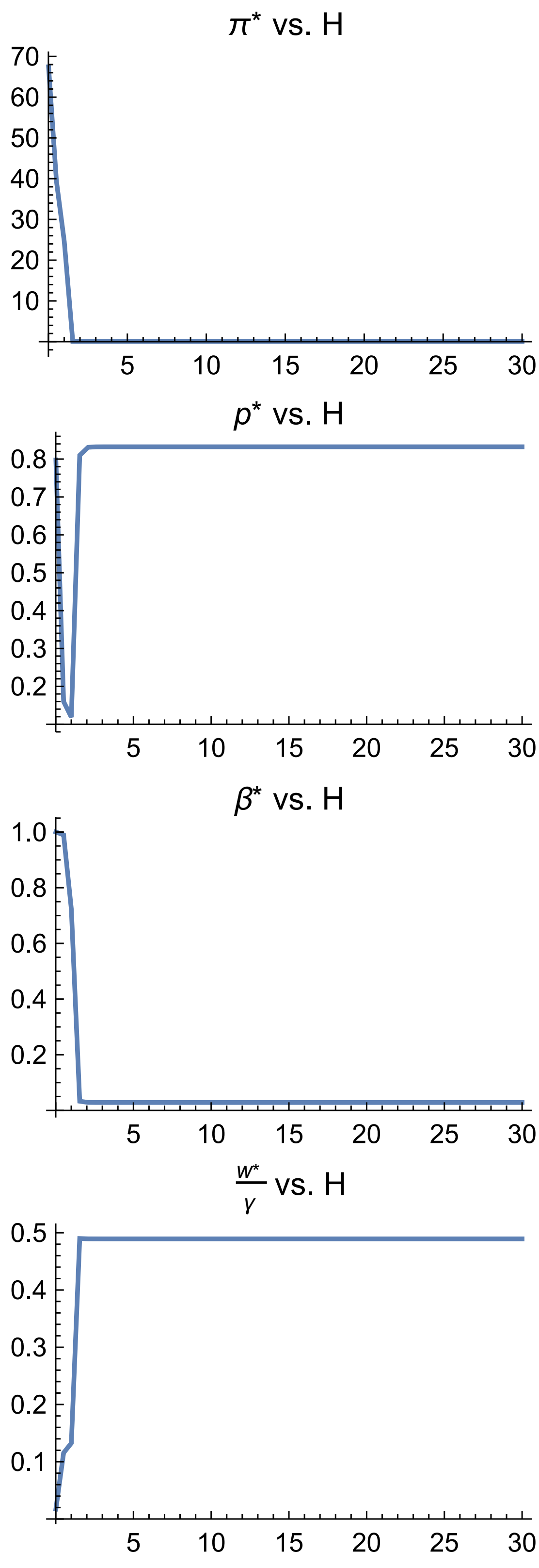


**Figure 4:** *Profit and optimal parameters when $\overline{\theta} = 8, \gamma = 2.25$, $c = 4.85$, and $\overline{L} = 30$.*

2. *$\beta^*$ is decreasing in $H$*

When $H = 0$, there is no risk of injury, and there is no need to invest in safety: $p^* = 1$ (certain injury). As $H$ increases, it is profitable for the firm to invest in safety (lower $p^*$) and raise wages to attract workers whose utility would otherwise be negative. However, eventually $H$ is sufficiently high ($H > \underline{H}$) that it is not profitable for the firm to attract any but the least risk-sensitive workers, for whom wages provide higher utility and response to injury magnitude $H$ is less pronounced. On this region, $p^*$ rises again to 1 as $H$ gets larger, and the firm is unable to profitably employ workers. This is depicted in Figure 4 and confirmed across a range of parameters.

At $H = 0$, it is profitable to employ the entire labor pool: they are all willing to work, and the firm's productive output under labor scarcity, equal to $\gamma L$, has no decreasing marginal return. As $H$ increases, however, a wage high enough to capture the most risk-averse workers is no longer profitable, and the optimal working fraction of the labor supply, $\beta^*$, decreases.

## 4.2 $\Pi^*$ vs Labor Pool Size $\overline{L}$ (and Cost of Safety $c$)

**Proposition 2.** *In addition to higher profit with higher $\overline{L}$, the following characterize the system response to the size of labor supply $\overline{L}$:*

1. *$p^*$ decrease in $\overline{L}$*
2. *$w^*$ decreases in $\overline{L}$*
3. *$\beta^*$ increases in $\overline{L}$*

Note that both $\overline{L}$ and $c$ are exogenous parameters, and so maximizing firm profit $\Pi$ in (5) is equivalent to maximizing $\frac{\Pi}{c}$. Written as such,

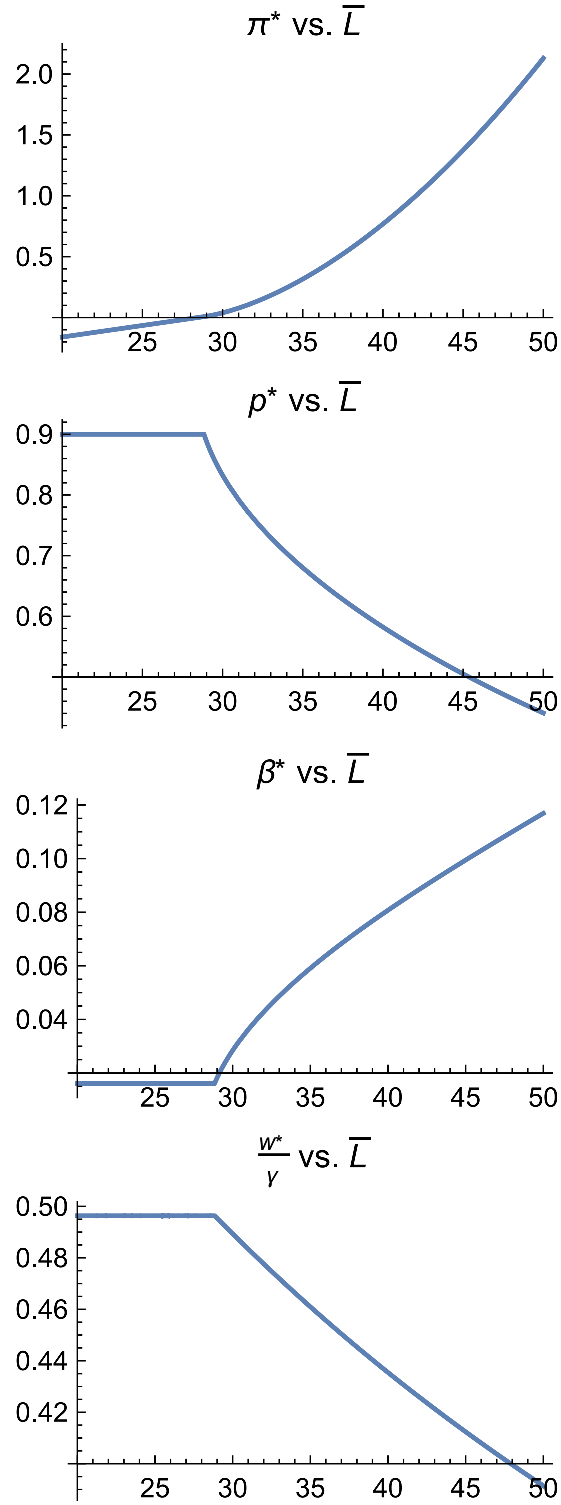


**Figure 5:** *Profit and optimal parameters when $\overline{\theta} = 8, \gamma = 2.25$, $c = 4.85$, and $H = 35$.*

$$\underset{p,\tilde{\theta}}{\text{Maximize}}\ \frac{\Pi}{c}$$
$$\frac{\overline{L}}{c}\cdot\frac{1}{\overline{\theta}}\left(\gamma\tilde{\theta}+\frac{1}{1-p}\ln\left(1-p\tilde{\theta}^2(1-e^{-\frac{H}{\tilde{\theta}}})\right)\right)-\frac{1-p}{p}$$
$$0\le p\le 1$$
$$0\le\tilde{\theta}\le\overline{\theta}$$

it is clear that the ratio $\frac{\overline{L}}{c}$ attenuates the tradeoff between net labor revenue (productive output minus wages paid) and safety investment ($C(1-p)=c\frac{1-p}{p}$). Seen in this way, increasing $\overline{L}$ reduces the relative cost of investing in safety. As a result, it is increasingly worthwhile to invest in a lower injury probability $p$.

By the same reasoning, it is less *relatively* cost-efficient to invest in wages versus safety. It is also less *absolutely* cost-efficient to invest in wages, and wages drop as $\overline{L}$ increases! The firm invests more in safety and less in wages as $\overline{L}$ increases.

Similarly, as $\overline{L}$ increases, it becomes increasingly profitable to capture a fixed fraction of the labor pool, and becomes more worthwhile to capture a larger fraction of the labor pool, increasing $\beta^*$.

### 4.3 $\Pi^*$ vs Maximum Risk-Response $\overline{\theta}$

**Proposition 3.** *In addition to profit decreasing as $\overline{\theta}$ increases, the following characterize the system response to the maximum risk-response parameter $\overline{\theta}$*

1. *Wage $w$ increases in $\overline{\theta}$*
2. *$p^*$ increases in $\overline{\theta}$*
3. *$\beta^*$ decreases in $\overline{\theta}$*

As $\overline{\theta}$ increases, risk-response across the population increases (recall a worker's risk-response

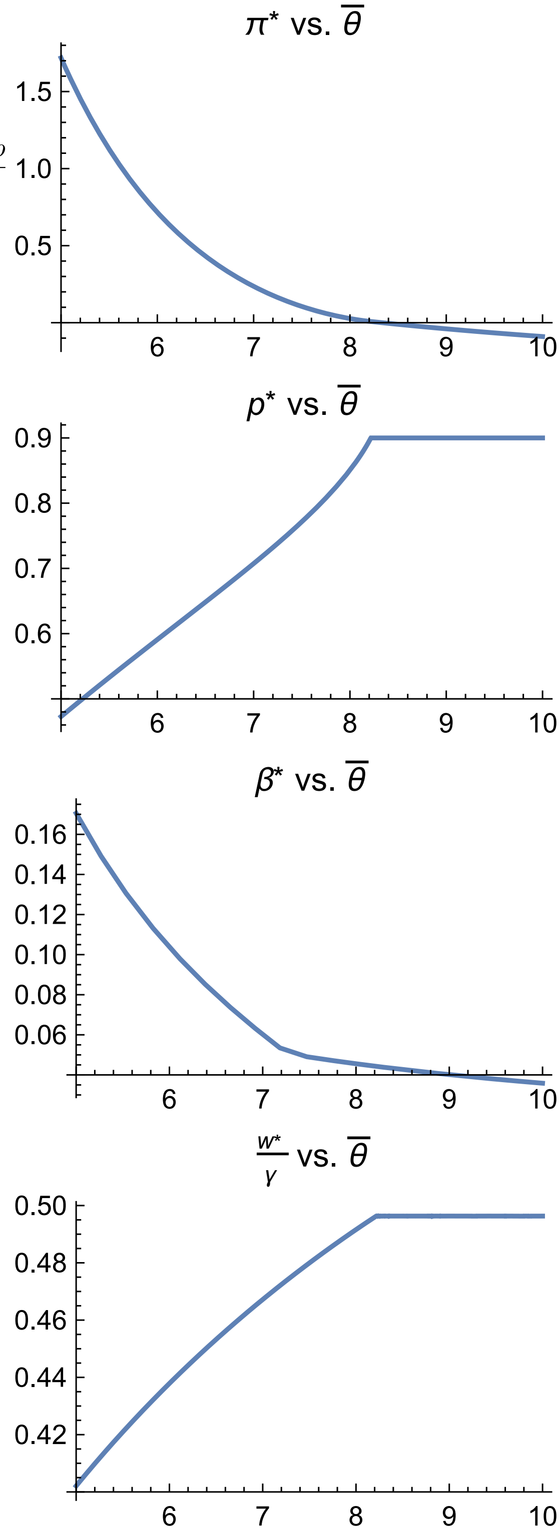


**Figure 6:** *Profit and optimal parameters when $\gamma = 2.25$, $c = 4.85$, $H = 35$, and $\overline{L} = 30$.*

$\theta \sim \text{Unif}[0, \overline{\theta}]$). This makes workers overall less sensitive to increases in their wage, and yet optimal wage rises in response. The reason is that as $\overline{\theta}$ increases, a higher wage is necessary to capture the same fraction of the labor pool. Recall that a higher risk-response $\theta$ does not just flatten the utility curve for income, but it strictly decreases it. Thus, it pays to pay workers more to keep them employed and $w^*$ increases. Note that in all the figures (including Figure 6), wage is illustrated via $\frac{w^*}{\gamma}$. This is because in the basic model $\Pi = (\gamma - w)L - c\frac{1-p}{p}$ it is the relative difference between $w$ and $\gamma$ that determines whether it is profitable to aim to capture more workers (by raising the wage or investing in more safety to increase $L$).

While it is profitable to increase wages to increase workers' utility as they become more risk-averse, it is not as profitable to increase their safety! This may be due to the modeling choice that risk-response to injury $\theta' = t\theta$ was simplified to $\theta' = \theta$ ($t = 1$) in section 3.1 and the choice of "cost of safety" $c\frac{1-p}{p}$ (which diverges to $\infty$ as $p \to 0$). This trend appeared consistently in numerical experiments, such as that depicted in Figure 6.

While wage increases in $\overline{\theta}$, and $p^*$ decreases, do more or fewer workers ultimately work as their risk-aversion increase? Predictably, the optimal fraction of workers $\beta^*$ decreases. This is because workers with higher risk-responses ($\theta$) are less willing to work and it is not profitable to increase wages so much as to keep $\beta^*$ constant (though wages *do* increase).

## 4.4 $\Pi^*$ vs Productivity $\gamma$

**Proposition 4.** *Worker productivity predictably increases firm profit, and the following characterize the system response to the worker productivity factor $\gamma$*

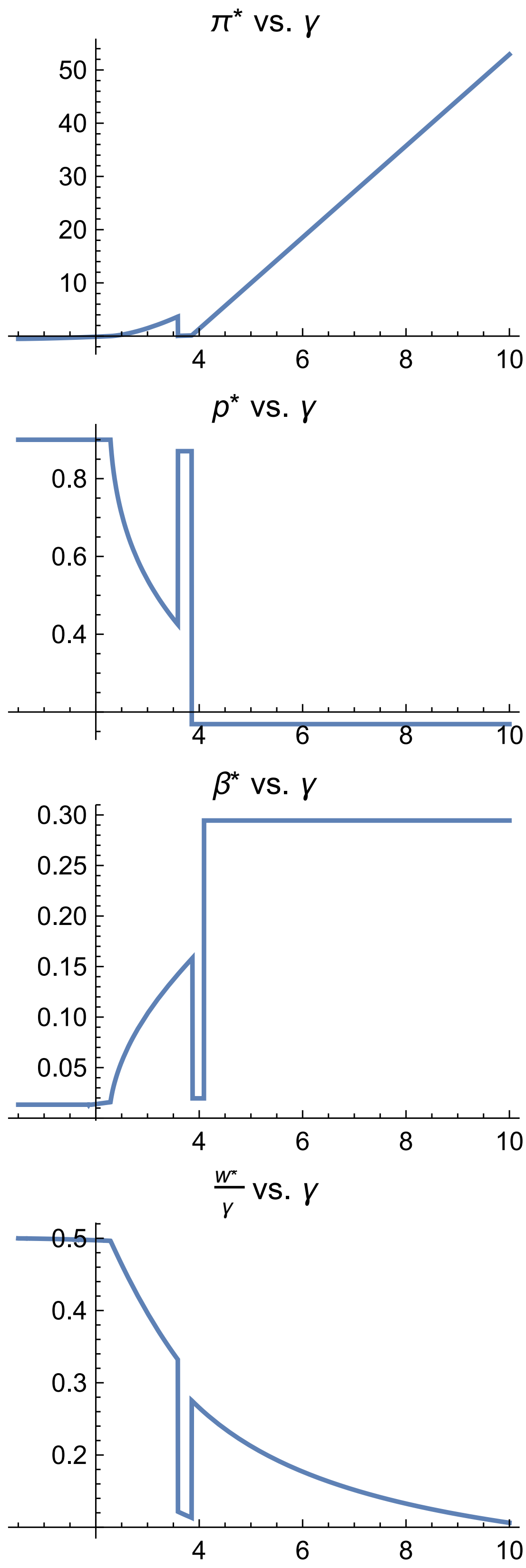


**Figure 7:** *Profit and optimal parameters when $\overline{\theta} = 8$, $c = 4.85$, $H = 35$, and $\overline{L} = 30$.*

1. *$\beta^*$ increases*
2. *$p^*$ decreases*
3. *$w^*$ decreases*

As worker productivity $\gamma$ increases, the firm finds it more profitable to employ more workers, and $\beta^*$ increases. However, in several numerical experiments, the firm does this by increasing safety: $p^*$ decreases, while wage $w^*$ actually falls. This can be seen in Figure 7 (which exhibits some numerical instability).

As described in 4.3, this is likely to some extent a by-product of the functional form of the cost of safety and the fact that workers' risk-response to injury and income is fixed somewhat arbitrarily. Still, this trend was exhibited in numerical experiments over several parameters.

# 5 Conclusion

This model explores the decisions of a firm investing in both wages and worker safety to attract workers under labor scarcity. During the COVID-19 pandemic, the need to invest in workers' safety has come into sharp focus as the need for medical personnel is heightened while, simultaneously, their willingness to work under dangerous conditions is tested.

This model extends basic worker income utility (with Constant Absolute Risk Aversion) to a more general "risk response" to both income and a cost of injury. The general risk response adheres to the idea from [15] that humans are risk-averse towards gains and risk-seeking towards losses (see Figure 1), and the risk-response parameter in our model reflects that some individuals who are more risk-averse with respect to income may also be more avoidant towards injury. Still, the core observation justifying a CARA utility function is that, empirically, risk-aversion seems to neither increase nor decrease in income; this paper assumes something comparable holds for injury response.

This model lays out a basic assumption on the cost of safety: zero safety ($p = 1$) costs nothing, and the cost of absolute safety ($p = 0$) diverges. In reality, firms face a menu of safety investment options, which combine (possibly non-additively) towards reducing the risk of any one type of injury; furthermore, there are many different types of injury, and no global "safety" level. Finally, workers respond heterogeneously to injury not just in their utility and willingness to work, but in the magnitude of the injury itself. In particular, this work considers workers' heterogeneity in risk-response, which makes some workers more disinclined to subject themselves to injury (higher disutility), but there may be an independent heterogeneous dimension contributing to this disutility that does not, for example, correlate with their response to wages.

This model is restricted to the case of labor scarcity, in which the firms yields non-decreasing marginal utility in workers. That is, for $L$ workers, productive output is $\gamma L$. A more realistic extension of this model would be a decreasing marginal utility in the labor force, but modeling the marginal utility of the firm's labor may not be a first-order concern for medical facilities. For example, many of the jobs in a hospital may be essential and the hospital cannot hire fewer than some fixed number of workers. Furthermore, workers may not be inclined to "not work" in the presence of several features of the real labor landscape. In particular, medical workers who have invested years into training, such as nurses, technicians, and doctors, command the relatively high wages (above the "going rate" for workers overall) that [20] describes as necessary to retain their employment; [20] also models how an equilibrium unemployment rate prevents turnover, and unemployment is a hallmark of the current pandemic.

Still, some of the results in Section 4 are unintuitive and surprising. As medical facilities and other employers face new challenges operating with an ever-present risk of injury in the long aftermath of a pandemic, it may be worth studying the dynamics of paying for worker safety and being profitable.

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