MACALESTER

MAXIMA REU in Interdisciplinary Mathematics

Anomaly Detection using Dictionary Learning

Mark Eisen[1], Mengjie Pan[2], Zachary Siegel[3] and Sara Staszak[4]

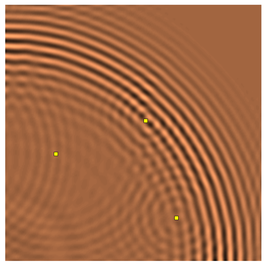
1: University of Pennsylvania, 2: Bryn Mawr College, 3: Pomona College, 4: Macalester College

TRANSFORMING THE WORLD IMA30 1982-2012 THROUGH MATHEMATICS

Introduction

Goals:

1. Find structural defects in wavefield data by detecting anomalous regions
2. Identify salient regions in natural images



Wavefield with 3 defects

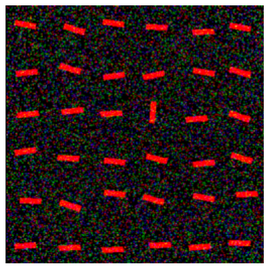


Image with visual saliency

Anomalous or salient regions of data exist in a different subspace than the majority. We apply dictionary learning and sparse coding to detect anomalies in various types of data by exploiting this property.

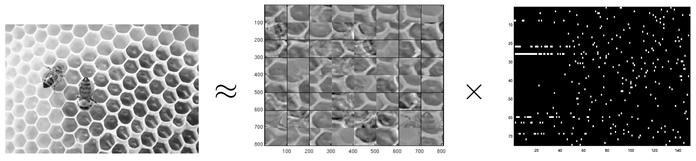
Dictionary Learning and Sparse Coding

Dictionary learning and sparse coding algorithms aim to solve:

$$\min_{D,X} ||Y - DX||_2^2 + \lambda ||X||_0 \tag{1}$$

where Y is the data, D is the dictionary that spans low-dimensional subspaces, X is the matrix of coefficients, and λ is the sparsity constraint.

$$\boldsymbol{Y} \approx \boldsymbol{D} \times \boldsymbol{X}$$



The two-step iterative method fixes D to generate X, then generates D and repeats.

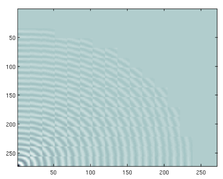
Detection Methods

We use three main anomaly detection methods:

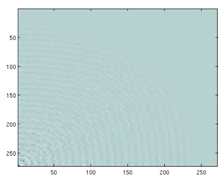
- **Residuals After Projection** A data point not well represented by sparse reconstruction will have a large residual. That is, if
$$\|Y_i - DX_i\|_2^2 > \tau, \tag{2}$$
where τ is an error threshold, then Y_i is an anomaly.
- **Influence and RANSAC** A data point that does not fit the model derived from a random sample of the data (itself not included) can be considered anomalous.
- **Unusual Dictionary Use** A data point that makes use of a 'rare' dictionary atom is anomalous where rare atoms are found by taking the row sums of absolute values in $X \cdot X^T$.

Extensions

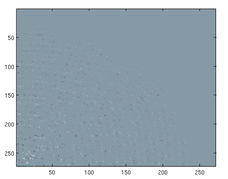
- **Choosing Dictionary Size and Numerical Dimension** 'Numerical rank' of data is the number of 'significant' singular values. This can be loosely extended to a 'sparse coding dimension,' as data is projected not onto one subspace but onto the union of subspaces. This rank informs choice of dictionary size.
- **Spatial Scoring** Directional orientation, as in time-oriented audio or spatially-oriented images, can be used to cross-validate predictions about anomalous behavior in a region.
- **Supervision** Using large training data sets to learn a dictionary reinforces classification of 'typical' and 'anomalous' data. This preprocessing is necessary when analyzing incomplete/undersampled data. Sucess at $< 10\%$ sampling (Shannon-Nyquist rate $\approx 2.5\%$).



100% Sampling



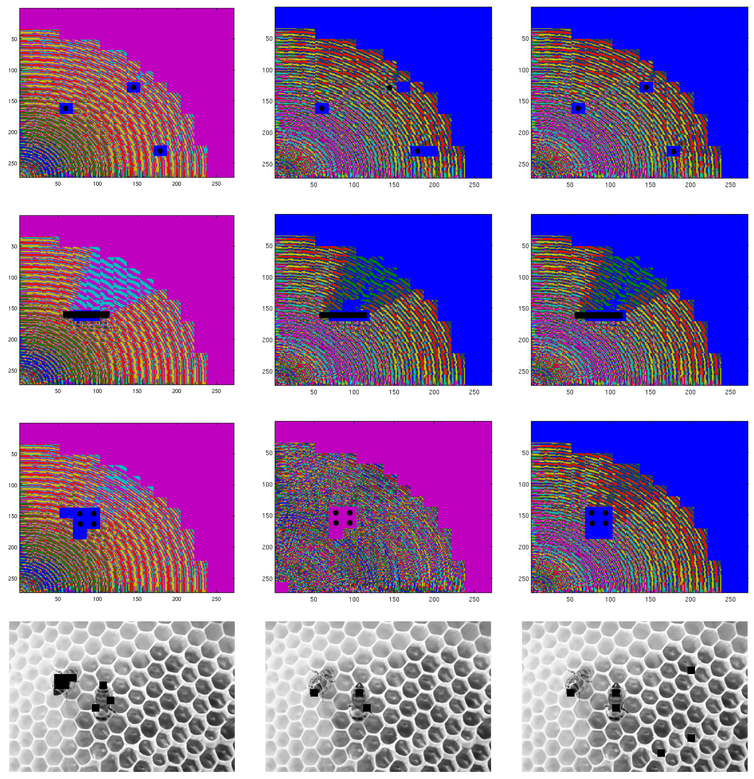
25% Sampling



5% Sampling

Results

Residual *RANSAC* *Rare*



References

- S. Gonella and J. Haupt, Automated defect localization via low rank plus outlier modeling of propagating wavefield data, submitted.
- I. Tošić and P. Frossard, Dictionary learning: what is the right representation for my signal? IEEE Signal Processing Magazine, March 2011.
- J. Marial, F. Bach, J. Ponce and G. Sapiro, Online dictionary learning for sparse coding, Proc. of the 26th Annual Int. Conf. on Machine Learning, 2009.
- M.A. Fischler and R.C. Bolles, Random sample consensus: a paradigm for model fitting with applications to image analysis and automated cartography, Commun. ACM, June 1981.

Faculty Advisor: Alicia Johnson (Macalester) **Problem Poser:** Jarvis Haupt (UMN)

Research Supported by an NSF REU grant and by the IMA